Kalinski and Eberly Reply: The authors of the Comment [1] conclude that since our *quantum* anti-Trojan system can be well approximated by the Hamiltonian of a pendulum [2] this implies that the classical dynamics is regular and therefore the localization around the unstable point has a purely classical origin. However, certain quantum systems are insensitive to the transition to chaos of the corresponding classical system [3]. The quantum states of such insensitive systems can be well approximated by the quantum pendulum even when the classical dynamics is chaotic.

The anti-Trojan is actually a canonical example. Our derivation of the pendular Hamiltonian was based on an approximate solution of the Schrödinger equation based on the properties of certain matrix elements [4] but not on the classical pendulum approximation quoted by the authors in Ref. [1]. In two dimensions, the Hamiltonian of our system (in the rotating frame using circular coordinates) is

$$H = \frac{p_r^2}{2} + \frac{p_{\phi}^2}{2r^2} - \frac{1}{r} + r\mathcal{E}\cos\phi - \omega p_{\phi}.$$
 (1)

It is the only constant of motion and the classical phase-space has a mixed structure: regular islands surrounded by stochastic layers (soft chaos) [5]. The regions covered by chaotic zones grow with the strength of the electric field  $\mathcal{E}$ .

The best confirmation of the quantum localization that we have mentioned [2] can be obtained by comparing a Poincaré section with the corresponding reduced Husimi distribution [6.] We have calculated the Husimi distribution for the anti-Trojan wave packet that we obtained from the pendulum approximation for  $m_0 = 19$ , with the scaled electric field below the critical condition given in our paper:  $\mathcal{T}_{sc} = 0.0064 < 3/4m_0$ . We have compared it with the corresponding Poincaré section in the  $(\phi, p_{\phi})$ plane for the same energy. In Fig. 1 the left graph shows the anti-Trojan probability distribution, localized around the unstable point (in radial coordinates around  $\phi = \pi$ ). The right graph superposes the corresponding Husimi distribution and Poincaré section. We note that the Poincaré section exhibits chaotic dynamics and the Husimi distribution covers the chaotic regions. This is sufficient evidence of quantum localization [7].

Another issue is our use of the term "scar." We mention that we are not the first to extend the term to such cases [7] following the first paper by Heller [8]. The scarring effect has a "purely quantum mechanical origin" in the sense that some eigenfunctions are supported by unstable periodic trajectories inside chaotic zones between residual invariant tori [6,7] but global stochasticity is not required [7]. Although supported by these prior examples, when some parts of the phase space contain regular structures there is no universal agreement yet about the boundaries of the terminology "purely quantum-mechanical," and so it need not be universally adopted [9].



FIG. 1. Probability density of anti-Trojan state for  $\mathcal{E}_{sc} = 0.0064$ ;  $\omega = 1/20^3$ , and (right) reduced Husimi distribution [6] at  $r = r_0 = (m_0 + 1)^2 = 20^2$  for this state superposed on the Poincaré section in  $(\phi, p_{\phi})$  coordinates. Small rectangular plots show one dimensional sections of the Husimi function for  $p_{\phi} = 20$  and  $\phi = \pi$ .

We thank the authors for showing numerically that our critical condition for existence of anti-Trojan states may be too severe. In our derivation we have used only sufficient conditions, and these may give unduly pessimistic estimates for localization, while remaining relevant to the issue of quantum scarring. We also thank the authors of the Comment for noticing the numerical mistake caused by flipping the sign of 1/3. This is almost obvious since both [10] and [11] predict the correct value  $\mathcal{E}_{stable} = (1 - q)q^{-1/3}$  for q = 8/9.

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