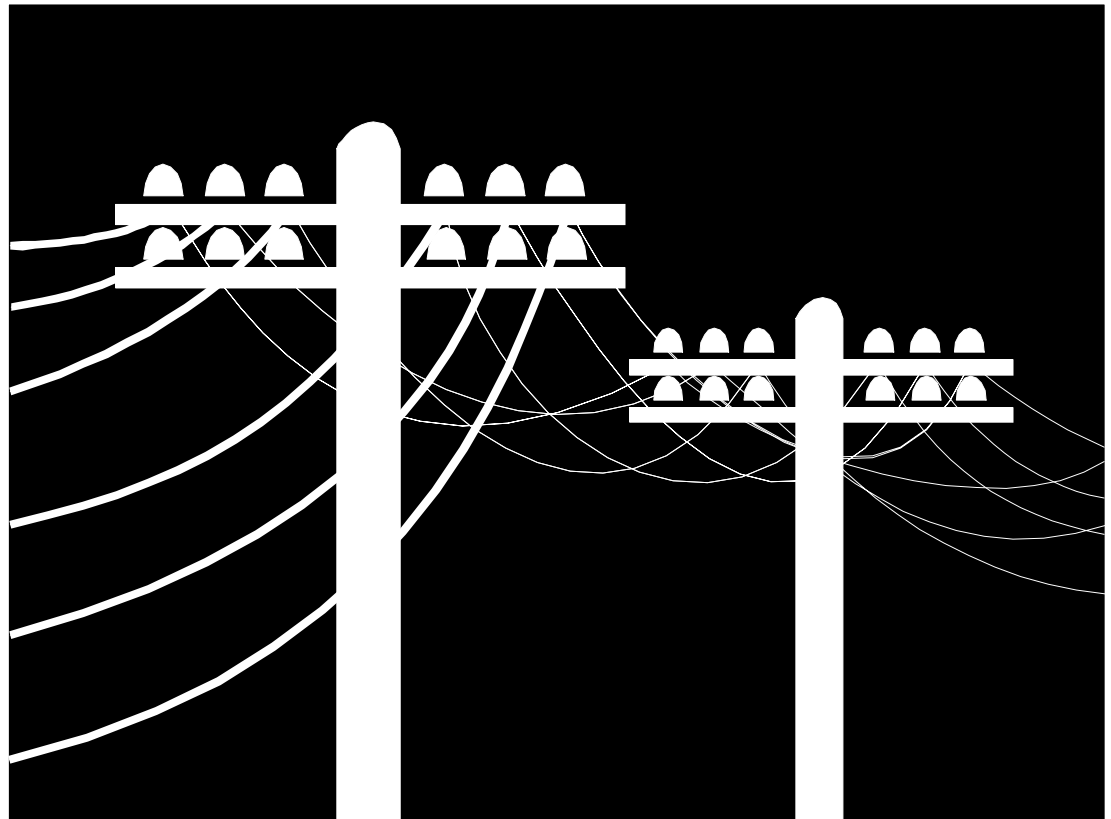

Today in Physics 217: conductors

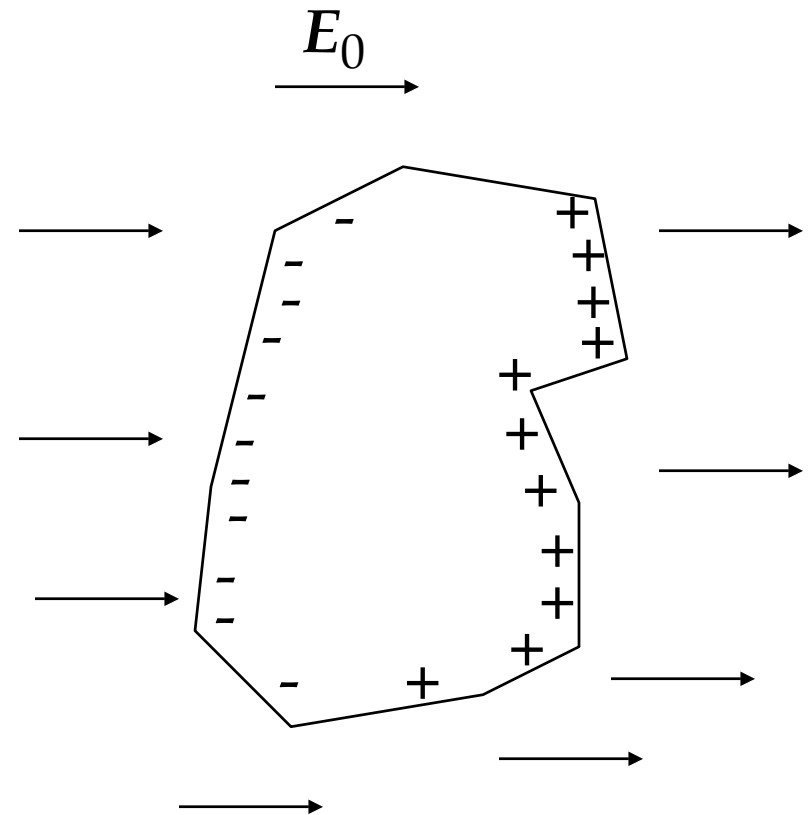
- ❑ Conductors: $E = 0$,
 $V = \text{constant inside}$.
- ❑ Examples
- ❑ Forces on
conductors from
surface charge



Conductors

Conductors are materials that contain charges that can move about freely (within the material) in response to an applied electric field.

- ❑ Metals are good examples; for *most* metals, the mobile carriers are electrons.
- ❑ Better conductors have higher densities of free (mobile) charges.



A metal in an externally-applied electric field. Electrons have moved to the “upstream” surface, leaving fixed positive charges behind.

Conductors (continued)

Implications of the free, mobile charges in conductors **in electrostatics**:

- **$E = 0$ inside a conductor.** If it weren't, charges would move; the free charges move whenever the field is nonzero, and stop only when $E = 0$ inside.
- **$\rho = 0$ inside a conductor.** This is because, for any surface completely within the conductor,

$$Q_{\text{enclosed}} = \frac{1}{4\pi} \oint E \cdot da = 0$$

because $E = 0$ there.

- **Any net charge lies on the surface.** That's as far as it can travel under the influence of the external electric field; also, there'd be nonvanishing E and ρ if this weren't true.

Conductors (continued)

- **V is constant throughout a conductor:** because, for any two points a and b inside,

$$V(b) - V(a) = -\int_a^b \mathbf{E} \cdot d\mathbf{l} = 0$$

Thus the surfaces of conductors are equipotentials.

- **E is perpendicular to the surface of conductors.**
Otherwise, free charges would flow along the surface until any parallel component were zero.

- Or think of the boundary conditions on E derived on Friday: here, $E = 0$ inside, and $E_{\parallel, \text{inside}} - E_{\parallel, \text{outside}} = 0$.
- This is why sharp-pointed conductors make good lightning rods.

What can we do with conductors?

Or, rather, what kinds of homework problems can I assign about conductors?

- Given conductors held at specified potentials, find the fields in between.
- Given conductors held at specified potentials, find the surface charge density.
- Given conductors immersed in specified electric fields, find the density of charge induced on the surface.
- Or vice-versa, for any of these.

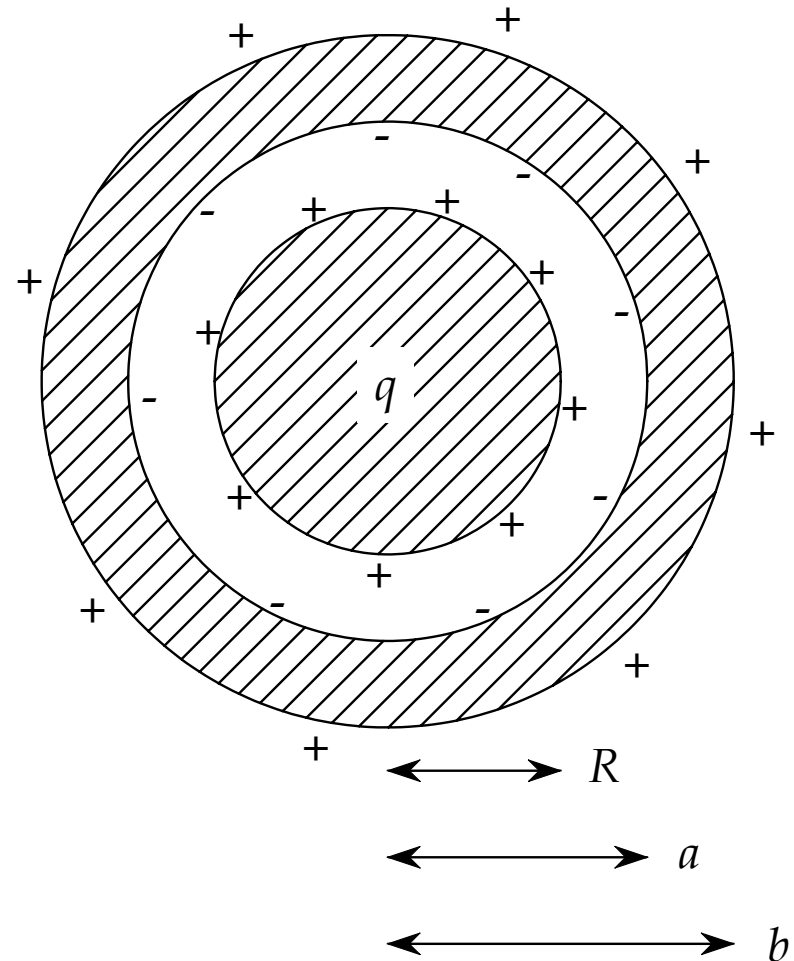
Example: concentric conducting spheres

Griffiths, problem 2.35: A metal sphere of radius R , carrying charge q , is surrounded by a thick concentric metal shell (inner radius a , outer radius b). The shell carries no net charge.

(a) Find the surface charge density, σ , on each surface.

(b) Find the potential at the center, using infinity as the reference point.

(c) Now the outer surface is **grounded**; that is, it is connected by a conducting wire to zero potential (same potential as at infinity). How do the answers to (a) and (b) change?



Example: concentric conducting spheres (continued)

(a) First, the inner sphere. A conductor carries its charge on its surface, so the density there is the total charge divided by total area:

$$\sigma_R = q / 4\pi R^2$$

Next, the inner surface of the shell. A charge is induced on this surface so as to generate an electric field equal and opposite to that from the sphere's charge, within the shell, so that $E = 0$ there. Spherical charged surfaces generate fields outside themselves that appear as if their charge is concentrated at the center, so the charge on the inner surface is $-q$:

$$\sigma_a = -q / 4\pi a^2$$

Example: concentric conducting spheres (continued)

Finally, the outer surface of the shell. The shell has no net charge, and $-q$ was induced on its inner surface, so there must be $+q$ there:

$$\sigma_b = q/4\pi b^2$$

(b) The field outside the shell is $E = \hat{r} q/r^2$, so the potential of its outer surface is $V = q/b$. This is also the potential at the inner surface, since conductors are equipotentials. The field between the inner surface of the shell and the sphere is $E = \hat{r} q/r^2$, so

$$V(R) = V(a) - \int_a^R \frac{q}{r^2} dr = \frac{q}{b} + \frac{q}{r} \Big|_a^R = \frac{q}{b} + \frac{q}{R} - \frac{q}{a}$$

Since conductors are equipotentials, this applies at the center of the sphere, too.

Example: concentric conducting spheres (continued)

(c) Now the shell is grounded. The field from the sphere's charge still has to be cancelled within the shell, so the surface charge densities on the sphere and on the inner surface of the shell are the same as before. But now the potential on the outer surface of the shell is zero, as at infinity, so the field must be zero outside the shell:

$$V(b) - V(\infty) = -\int_b^{\infty} \mathbf{E} \cdot d\mathbf{l} = 0 \quad \Rightarrow \quad \mathbf{E} = 0$$

and thus there's no charge on this surface: $\sigma_b = 0$.

And since the potential is zero on the inner surface too,

$$V(R) = V(a) - \int_a^R \frac{q}{r^2} dr = \frac{q}{r} \Big|_a^R = \frac{q}{R} - \frac{q}{a}$$

Example: field in an empty cavity within a conductor

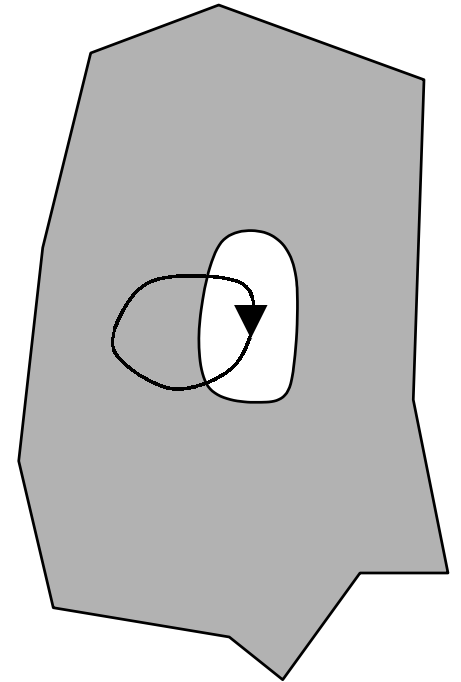
By empty, we mean there's no charge there. What is the field there?

Consider a loop lying partly within the cavity, and partly within the conductor. $E = 0$ in the conductor, and $\oint E \cdot dl = 0$, so

$$\int_{\text{path thru cavity}} E \cdot dl = 0$$

Since the path is finite, the field within the cavity must be zero.

This is why you can shield external electrostatic fields with a conducting box (Faraday cage), and why you won't get electrocuted if lightning strikes your car while you're inside. (Unless you drive a Saturn or a Corvette, that is.)



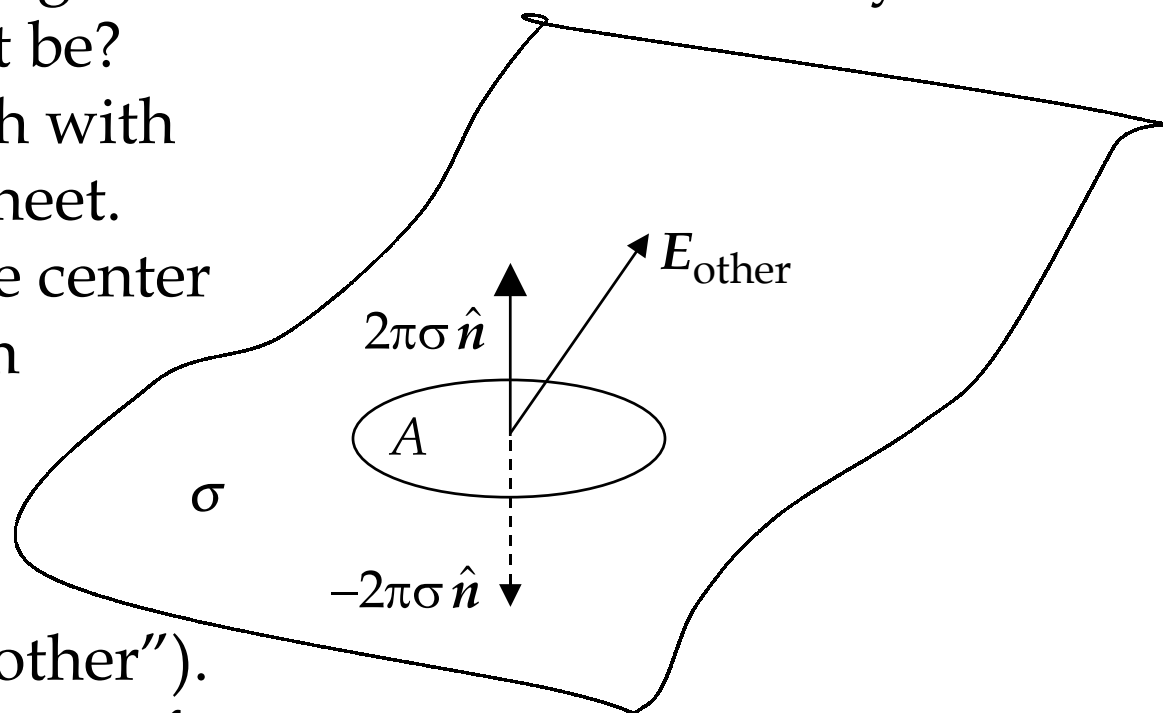
Forces on conductors

If there's a field difference across a surface charge, as there is on the surface of a charged conductor, then there may be a force, too. What will it be?

Consider a small patch with area A on a charged sheet.

The total field near the center of the patch is the sum of the patch's field and the field from the rest of the charge distribution ("other").

Only the "other" E exerts a force on the patch. What is E_{other} ?



Forces on conductors (continued)

Just above and just below the center of the patch, we have

$$E_{\text{above}} = E_{\text{other}} + 2\pi\sigma \hat{n}$$

$$E_{\text{below}} = E_{\text{other}} - 2\pi\sigma \hat{n}$$

$$E_{\text{above}} + E_{\text{below}} = 2E_{\text{other}} \quad \Rightarrow \quad E_{\text{other}} = (E_{\text{above}} + E_{\text{below}})/2$$

We can also recall the boundary condition derived on Friday,

$$E_{\text{above}} - E_{\text{below}} = 4\pi\sigma \hat{n}$$

So, for instance, the field just above a charged conductor is $E = 4\pi\sigma \hat{n}$, the field below the surface is zero, and

$$F = qE_{\text{other}} = \sigma A \frac{1}{2} (4\pi\sigma \hat{n} + 0) = 2\pi\sigma^2 A \hat{n}$$

Forces on conductors (continued)

Or, to express this as a pressure (force per unit area) in terms of the field just outside the conductor,

$$P = \frac{F}{A} = 2\pi\sigma^2 = \frac{1}{8\pi} E_{\text{above}}^2$$