Quantum Mechanics II - Module 1

Welcome to the PHY246 recitation! The purpose of the recitation is to review some of the more challenging aspects of the course, as well as explore some areas that you were unable to tackle in class due to time restrictions. Throughout the recitation, you will work on some problems individually, some problems in small groups, and occasionally you will tackle a problem as a large group. Studies have shown that learning is enhanced when students work together, so let the learning begin! Take a moment to introduce yourself and meet everyone else in the recitation.

Postulates of Quantum Mechanics

Spend a few minutes answering the following true/false questions individually. The TA will indicate when time is up and then you will discuss your answers as a large group.

1. True or false: Eigenvalues may be discrete or continuous.
2. True or false: The operator $\hat{R}$, which acts on a function $\phi(x)$ by $\hat{R}\phi(x) = \phi(x)^3 + 1$ is linear.
3. A free particle is moving in one dimension. It is not known what state the particle is in and a measurement of the particle's momentum yields $p = \hbar k$. True or false: Another measurement, made immediately after the first measurement, will not necessarily yield $p = \hbar k$ for the momentum.
4. True or false: For any function $f(x)$, the following is true: $f(x)\delta(x) = f(0)\delta(x)$.
5. True or false: A measure of the relative uncertainty in the position of a particle moving in one dimension is given by the square root of the variance.
6. True or false: The expectation of any observable is always constant in time.

Gaussian Integral Refresher

Break up into groups of three or four people (the TA can help you form groups if necessary) and attempt to compute the following integrals. If you are unfamiliar with how to handle Gaussian integrals, ask a friend, or ask the TA.

1. $\int_{-\infty}^{+\infty} e^{-x^2}\, dx$
2. $\int_{-\infty}^{+\infty} xe^{-x^2}\, dx$
3. $\int_{-\infty}^{+\infty} x^2e^{-x^2}\, dx$

More Fun with the Quantum Harmonic Oscillator

On your homework you are asked to consider the system with Hamiltonian

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2.$$

Suppose that this system has a normalized wave function given by

$$\psi(x, t = 0) = (A + Bx)e^{-\alpha x^2}$$

where α is determined as in your homework. (If you haven't figured out how to determine α yet, ask your TA!)

1. What is the relation between A and B? (Hint: Normalize!)
2. How do you add time dependence to a wave function? What is $\psi(x, t)$ for $t > 0$?
3. Let $\langle\hat{x}\rangle_t$ be the expectation value of x with time dependence included. What are $\langle\hat{x}\rangle_t$ and $\langle\hat{p}\rangle_t$?