Quantum Mechanics II - Module 2

Dirac Notation

- 1. Express $\langle \phi | \psi \rangle$ as an integral.
- 2. Express $\langle \phi | p | \psi \rangle$ as an integral.
- 3. For a discrete set of states, what is the orthonormality condition in Dirac notation?
- 4. For a discrete set of states, what is the completeness condition in Dirac notation?

Hilbert Space

An electron in a one-dimensional box with walls at x = 0 and x = a is in the quantum state

$$\psi(x) = Ax(x-a).$$

- 1. Using only three or four sentences, how would you describe what a Hilbert space is to someone who has an introductory knowledge of quantum mechanics?
- 2. Find the normalization constant A.
- 3. Represent the system as a ket vector $|\psi\rangle$ in the Hilbert space spanned by the eigenstates of the Schrödinger equation for a particle in a box.
- 4. What are the possible outcomes of the energy measurements? What are their probabilities?
- 5. What is the expectation value of energy in the state $|\psi\rangle$?

Lorentzian Wave Function

Consider a wave function of Lorentzian form, i.e.,

$$\psi(x) = \frac{1}{x^2 + a^2}.$$

- 1. What are $\langle x \rangle$ and Δx ?
- 2. What are $\langle p \rangle$ and Δp ?
- 3. Compare with the uncertainty relation. Show from general considerations that the uncertainty product is necessarily independent of a.