Quantum Mechanics II - Module 3

Commutator Concepts

1. Explain what it means to say that two Hermitian operators, A and B, satisfy $[A, B] = 0$. What is the physics that is contained in this statement? Be sure to discuss the case when one (or both) of the operators is degenerate.

2. What is meant by a "complete set of commuting operators?"

Hermitian Operators

1. Given that the operators A and B are Hermitian, what can you say about each of the following?

   (a) AB

   (b) $AB + BA$

   (c) $[A, B]$

   (d) $i[A, B]$

2. Consider the following equation consisting of kets, scalars, and operators (C, D, not necessarily Hermitian):

$$\alpha_1|V_1\rangle = \alpha_2|V_2\rangle + \alpha_3|V_3\rangle\langle V_4|V_5\rangle + \alpha_4 CD|V_6\rangle.$$

   What is its adjoint?

3. Suppose that A is a Hermitian operator satisfying the following eigenvalue equation:

$$A|\omega\rangle = \omega|\omega\rangle.$$

   Show that the eigenvalues ω are real.

Trial Wave Functions

Consider the simple harmonic oscillator with

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2.$$

1. Let $\psi_1(x) \sim e^{-\alpha|x|}$ where $\alpha > 0$ is a variational parameter. Compute $\langle H\rangle$ as a function of α.

2. Minimize this quantity with respect to α and determine the value of $\langle H \rangle$ at the minimum. To what exact energy eigenvalue should this be considered an approximation?

3. Now let $\psi_2(x) \sim xe^{-\beta|x|}$ where $\beta > 0$ is another variational parameter. Compute $\langle H \rangle$ as a function of β.

4. Minimize this quantity with respect to β and determine the value of $\langle H \rangle$ at the minimum. To what exact energy eigenvalue should this be considered an approximation?

5. Show that $\psi_1(x)$ and $\psi_2(x)$ are orthogonal.