Quantum Mechanics II - Module 4

Conservation Theorems

1. Why are conserved quantities important?
2. What is a stationary state? What can you say about the expectation value of an operator (which does not contain time explicitly) in a stationary state?
3. What is Ehrenfest's principle?
4. What is parity?
5. Consider the infinite square well centered at the origin. Draw some pictures of some wavefunctions with even parity and draw some with odd parity.

Bound States of a Delta Function Potential

One kind of potential that you will (unfortunately) not encounter in your textbook is the delta function potential. This potential can (fortunately) be handled using the same ideas that you use to solve other one-dimensional potential problems. Delta function potentials admit both bound states and scattering states, so we will look at each one separately. First, consider a particle of mass m in a delta function potential given by $V(x) = -\alpha\delta(x)$ where α is some constant and consider only bound states ($E < 0$).

1. What is the form of the wavefunction for $x < 0$?
2. What is the form of the wavefunction for $x > 0$?
3. The wavefunction is continuous at the boundary. Apply this boundary condition to the forms of the wavefunction that you found above.
4. Normally, the second boundary condition would be "$d\psi/dx$ is continuous at the boundary," but that only applies when the potential is not infinite at the boundary. To derive the equivalent boundary condition for this case, integrate the Schrodinger equation from $-\epsilon$ to $+\epsilon$ and then take the limit as $\epsilon \to 0$. You should get the following condition:

$$\kappa = \frac{m\alpha}{\hbar^2}.$$

5. Finally, normalize your wavefunction and show that the delta function potential has exactly one bound state, regardless of its "strength" α.

Scattering States of a Delta Function Potential

Now consider a particle of mass m in a delta function potential given by $V(x) = -\alpha\delta(x)$ where α is some constant and consider only scattering states $(E > 0)$. Also, assume that the particle is approaching from the left.

1. What is the form of the wavefunction for $x < 0$?

2. What is the form of the wavefunction for $x > 0$?

3. Apply the boundary conditions to the wavefunctions that you found above.

4. Calculate the reflection and transmission coefficients and show that $R + T = 1$. (See the discussion on delta function potentials in "Introduction to Quantum Mechanics" by David J. Griffiths for a discussion on some subtle points regarding the transmission and reflection coefficients that I am glossing over here.)