0.1. **LastExam.**

$$\mathbf{E}(\mathbf{r}) = -\nabla\Phi(\mathbf{r})$$

$$\Phi(\mathbf{r}) = \int d^3r' \frac{\rho(\mathbf{r'})}{|\mathbf{r}-\mathbf{r}'|}$$

$$\Phi(\mathbf{r}) = -\int_{\infty}^{\mathbf{r}} d\mathbf{l}\bullet\mathbf{E}$$

$$\sigma = \frac{1}{4\pi}\hat{\mathbf{n}}\bullet(\mathbf{E_r}-\mathbf{E_l})$$

0.2. **Maxwell's Equations.**

$$\nabla\bullet\mathbf{D} = 4\pi\rho$$

$$\nabla\bullet\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{H} = \frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla^2\Phi - \frac{\epsilon\mu}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{4\pi}{\epsilon}\rho$$

$$\nabla^2\mathbf{A} - \frac{\epsilon\mu}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{\epsilon}\mathbf{J}$$

0.3. **Solutions to Laplace Equation. Cartesian**

$$X''(x) = \alpha_1 \quad Y''(y) = \alpha_2 \quad Z''(z) = \alpha_2 \quad \alpha_1+\alpha_2+\alpha_3 = 0$$

Solve the equation which has some non-zero potential B.C. last.

$$\Phi(x,y,z) = \sum_{m,n=1}^{\infty} A_{m,n,k}(X_m(x)Y_n(y)Z_k(z))$$

Spherical (azimuthal symmetry)

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{\infty}(A_l r^l + B_l r^{-(l+1)})P_l(cos(\theta))$$

Cylindrical (no z dependence)

$$\Phi(r,\phi) = C_0+D_0 ln(r)+\sum_{n=1}^{\infty}(C_n r^n + D_n r^{-n})(A_n cos(n\phi)+B_n sin(n\phi))$$

0.4. **math.**

$$|\mathbf{r}-\mathbf{r}'| = \sqrt{r^2+r'^2-2rr'cos(\theta')}$$

$$\nabla\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|^n}\right) = -\frac{n}{|\mathbf{r}-\mathbf{r}'|^{n+2}}(\mathbf{r}-\mathbf{r}') \quad n >= 1$$

$$\nabla^2\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) = -4\pi\delta^3(\mathbf{r}-\mathbf{r}')$$

$$P_l(x) = \frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l$$

0.5. **Vector Calculus.**

$$\mathbf{A}\bullet(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\bullet(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\bullet(\mathbf{A}\times\mathbf{B})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\bullet\mathbf{C}) - \mathbf{C}(\mathbf{A}\bullet\mathbf{B})$$

$$\nabla(fg) = f(\nabla g)+g(\nabla f)$$

$$\nabla(\mathbf{A}\bullet\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B})+\mathbf{B}\times(\nabla\times\mathbf{A})+(\mathbf{A}\bullet\nabla)\mathbf{B}+(\mathbf{B}\bullet\nabla)\mathbf{A}$$

$$\nabla(f\mathbf{A}) = f(\nabla\bullet\mathbf{A})+\mathbf{A}\bullet(\nabla f)$$

$$\nabla\bullet(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\bullet(\nabla\times\mathbf{A}) - \mathbf{A}\bullet(\nabla\times\mathbf{B})$$

$$\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\bullet\nabla)\mathbf{A} - (\mathbf{A}\bullet\nabla)\mathbf{B} + \mathbf{A}(\nabla\bullet\mathbf{B}) - \mathbf{B}(\nabla\bullet\mathbf{A})$$

$$\nabla\bullet(\nabla\times\mathbf{A}) = 0$$

$$\nabla\times(\nabla f) = 0$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\bullet\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\int_a^b(\nabla f)\bullet d\mathbf{l} = f(\mathbf{a}) - f(\mathbf{b})$$

$$\int(\nabla\bullet\mathbf{A})d\tau = \oint\mathbf{A}\bullet d\mathbf{a}$$

$$\int(\nabla\times\mathbf{A})\bullet d\mathbf{a} = \oint\mathbf{A}\bullet d\mathbf{l}$$

$$\int_V d^3r'(A\nabla'^2B - B\nabla'^2A) = \int_S d\mathbf{s}'\bullet(A\nabla'B - B\nabla'A)$$

0.6. **Coordiates.**

$x = rsin(\theta)cos(\phi) \quad y = rsin(\theta)sin(\phi) \quad z = rcos(\theta)$

$\theta\in[0,\pi] \quad \phi\in[0,2\pi]$

$x = s\ cos(\phi) \quad y = s\ sin(\phi) \quad z = z$

$$d\mathbf{l} = dx\hat{\mathbf{x}}+dy\hat{\mathbf{y}}+dz\hat{\mathbf{z}}\ ; d\tau = dx\,dy\,dz$$

$$d\mathbf{l} = dr\hat{\mathbf{r}}+rd\theta\hat{\theta}+rsin(\theta)\hat{\phi}\ ; d\tau = r^2 sin(\theta)dr\,d\theta\,d\phi$$

$$d\mathbf{l} = ds\hat{\mathbf{s}}+sd\phi\hat{\phi}+dz\hat{\mathbf{z}}\ ; d\tau = s\,ds\,d\phi\,dz$$

$$\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}}+\frac{\partial t}{\partial y}\hat{\mathbf{y}}+\frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

$$\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}}+\frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta}+\frac{1}{rsin(\theta)}\frac{\partial t}{\partial\phi}\hat{\phi}$$

$$\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}}+\frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\phi}+\frac{\partial t}{\partial z}\hat{\mathbf{z}}$$

$$\nabla\bullet\mathbf{v} = \frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}$$

$$\nabla\bullet\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)+\frac{1}{rsin(\theta)}\frac{\partial}{\partial\theta}(sin(\theta)v_\theta)+\frac{1}{rsin(\theta)}\frac{\partial v_\phi}{\partial\phi}$$

$$\nabla\bullet\mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(sv_s)+\frac{1}{s}\frac{\partial v_\phi}{\partial\phi}+\frac{\partial v_z}{\partial z}$$

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r})\frac{1}{r^2 sin(\theta)}\frac{\partial}{\partial\theta}(sin(\theta)\frac{\partial t}{\partial\theta})+\frac{1}{r^2 sin^2(\theta)}\frac{\partial^2 t}{\partial\phi^2}$$

$$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}(s\frac{\partial t}{\partial s})+\frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2}+\frac{\partial^2 t}{\partial z^2}$$

0.7. trig.

$$sin(u \pm v) = sin(u)cos(v) \pm cos(u)sin(v)$$
$$cos(u \pm v) = cos(u)cos(v) \mp sin(u)sin(v)$$
$$sin(2u) = 2sin(u)cos(u)$$
$$cos(2u) = cos^2(u) - sin^2(u) = 1 - 2sin^2(u)$$
$$sin^2(u) = \frac{1 - cos(2u)}{2}$$
$$cos^2(u) = \frac{1 + cos(2u)}{2}$$
$$sin(u) + sin(v) = 2sin(\frac{u+v}{2})cos(\frac{u-v}{2})$$
$$sin(u) - sin(v) = 2cos(\frac{u+v}{2})sin(\frac{u-v}{2})$$
$$cos(u) + cos(v) = 2cos(\frac{u+v}{2})cos(\frac{u-v}{2})$$
$$cos(u) - cos(v) = -2sin(\frac{u+v}{2})sin(\frac{u-v}{2})$$
$$sin(u)sin(v) = \frac{1}{2}(cos(u-v) - cos(u+v))$$
$$cos(u)cos(v) = \frac{1}{2}(cos(u-v) - cos(u+v))$$
$$sin(u)cos(v) = \frac{1}{2}(sin(u+v) - sin(u-v))$$
$$cos(u)sin(v) = \frac{1}{2}(sin(u+v) - sin(u-v))$$

2

0.8. Green's Functions.

$$\Phi(\mathbf{r}) = \frac{1}{4\pi}\int_s \left(G(\mathbf{r},\mathbf{r}')\frac{\partial\Phi(\mathbf{r}')}{\partial n'} - \Phi(\mathbf{r}')\frac{\partial G(\mathbf{r},\mathbf{r}')}{\partial n'}\right) + \int_V d^3r' G(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')$$

0.8.1. *Direchelete (known on surface, and require $G_D(\mathbf{r},\mathbf{r}') = 0 \ for \ \mathbf{r} \ or \ \mathbf{r}' \in S$).*

$$\nabla^2 G(\mathbf{r},\mathbf{r}') = -4\pi\delta(\mathbf{r},\mathbf{r}')$$
$$\Phi(\mathbf{r}) = -\frac{1}{4\pi}\int_s \Phi(\mathbf{r}')\frac{\partial G_D(\mathbf{r},\mathbf{r}')}{\partial n'} + \int_V d^3r' G_D(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')$$
$$G(\mathbf{r},\mathbf{r}') = \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right) + H(\mathbf{r},\mathbf{r}')$$
$$\nabla^2 H(\mathbf{r},\mathbf{r}') = 0$$

0.9. BV in Space.

$$\mathbf{E}_{t,R} = \mathbf{E}_{t,L}$$
$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} = \epsilon\mathbf{E}$$
$$\epsilon_R\frac{\partial\Phi_R}{\partial n} = \epsilon_L\frac{\partial\Phi_L}{\partial n} + 4\pi\sigma$$
$$\mathbf{B}_{R,n} = \mathbf{B}_{L,n}$$
$$\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M} = \frac{\mathbf{B}}{\mu}$$
$$H_{R,t} = H_{L,t} + \frac{4\pi}{c}J_\sigma$$

0.10. Magnetostatics.

$$\mathbf{F_m} = \frac{q}{c}\mathbf{v}\times\mathbf{B}$$
$$\mathbf{J} = \rho\mathbf{v}$$
$$I = \int_s \mathbf{s}\bullet\mathbf{J} \simeq |\mathbf{J}|A \text{ if thin}$$
$$\frac{\partial\rho}{\partial t} + (\nabla\bullet\mathbf{J}) = 0$$
$$\mathbf{A}(\mathbf{r}) = \frac{1}{c}\int d^3r' \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$
$$\mathbf{F_1} - \frac{I_1 I_2}{c^2}\oint\oint \frac{d\mathbf{l_1}\times(d\mathbf{l_2}\times(\mathbf{r_1}-\mathbf{r_2}))}{|\mathbf{r_1}-\mathbf{r_2}|^3}$$
$$\mathbf{B}(\mathbf{r}) = \frac{I}{c}\oint \frac{d\mathbf{l}'\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r_1}-\mathbf{r_2}|^3}$$

0.11. relativity.

$$\gamma = \sqrt{1-\beta^2}$$
$$x^\mu = (ct,\mathbf{x}) = (ct,x,y,z)$$
$$x_\mu = (ct,-\mathbf{x}) = (ct,-x,-y,-z)$$
$$p^\mu = \left(\frac{E}{c},\mathbf{p}\right)$$
$$p_\mu = \left(\frac{E}{c},-\mathbf{p}\right)$$
$$p^2 = \eta_{\mu\nu}p^\mu p^\nu = \frac{E^2}{c^2} - \mathbf{p}^2$$
$$p^\mu = mu^\mu$$

0.12. EM-Waves.

$$E(\mathbf{x},t) = \mathbf{E^o}exp(\pm i\omega t + \mathbf{k}\bullet x)$$
$$\mathbf{S} = \frac{c}{2\pi}\mathbf{E}\times\mathbf{B}$$
$$P_{rad} = \int_V \nabla\bullet\mathbf{S}dV = \int_S d\mathbf{s}\bullet\mathbf{S}$$

2

1. FOUIER

$$\int_{-1}^{1} P_l(x)P_{l'}(x)dx = \int_0^{\pi} P_l(cos(\theta))P_{l'}(cos(\theta))sin(\theta)d\theta = \frac{2}{2l+1} \forall l = l'$$

$$\int_0^{2\pi} cos(a\phi)cos(b\phi) = 0 \ \forall a \neq b \ \ and = \pi \ \forall \ a = b$$

$$\int_0^{2\pi} sin(a\phi)sin(b\phi) = 0 \ \forall a \neq b \ \ and = \pi \ \forall \ a = b$$