

Estimate the standard deviation for each of the following rate measurements

<u>Counts</u>	<u>Duration (min)</u>	<u>Rate (/min)</u>	<u>Sigma</u>
210	5000	0.042	0.003
280	6000	0.047	0.003
500	10000	0.050	0.002
1100	20000	0.055	0.002
20	500	0.040	0.009
49	1000	0.049	0.007

Estimate the standard deviation for each of the following flux measurements

<u>Counts</u>	<u>Duration (min)</u>	<u>Paddle characteristics</u>			<u>Flux(/m²/s/sr)</u>	<u>Sigma</u>
		<u>Separation (cm)</u>	<u>Area 1 (cm²)</u>	<u>Area 2 (cm²)</u>		
500	5000	70	400	400	0.510	0.023
250	5000	100	400	400	0.521	0.033
100	5000	150	400	400	0.469	0.047
130	10000	200	400	400	0.542	0.048
160	5000	150	400	650.25	0.461	0.036
180	10000	200	400	650.25	0.461	0.034

Pick several pairs of either rate or flux measurements and work out the number of standard deviation difference between the measurements

Compare the 1st and 4th rate measurements:
the difference in rates and the standard deviation on the difference is:

<u>Difference</u>	<u>Sigma</u>
-0.013	0.003

Assume all these trials measure the same true rate.

- 1) Calculate the average rate over all these trials by summing counts and duration to get a total number of counts and duration and divide the two. (Think of this as combining the data into one big experiment.)
- 2) Calculate the number of standard deviations (Nsd) each trial departs from this average
- 3) Highlight any differences among measurements that are "significant"

Counts	Duration (min)	Rate (/min)	Sigma	Nsd
210	5000	0.042	0.003	-3.0
280	6000	0.047	0.003	-1.5
500	10000	0.050	0.002	-0.4
1100	20000	0.055	0.002	2.5
20	500	0.040	0.009	-1.2
49	1000	0.049	0.007	-0.3

Average	Sigma
0.0508	0.0011

Assume all these trials measure the same true flux.

- 1) Calculate the average flux over all these trials by taking a WEIGHTED AVERAGE
- 2) Calculate the number of standard deviations (Nsd) each trial departs from this average
- 3) Highlight any differences among measurements that are "significant"

Paddle characteristics								Flux*wt	wt
Counts	Duration	Separation	Area 1	Area 2	Flux	Sigma	Nsd		
500	5000	70	400	400	0.510	0.023	0.6	980	1919
250	5000	100	400	400	0.521	0.033	0.7	480	922
100	5000	150	400	400	0.469	0.047	-0.6	213	455
130	10000	200	400	400	0.542	0.048	0.9	240	443
160	5000	150	400	650.25	0.461	0.036	-1.0	347	752
180	10000	200	400	650.25	0.461	0.034	-1.0	390	846

Average	Sigma
0.497	0.014

WEIGHTED AVERAGE: An average is simply the sum of a series of measurements, X_i , divided by the number of measurements, N . In a WEIGHTED AVERAGE, each measurement carries a weight w_i . The weighted average is just the $[\text{sum of } (X_i * w_i)] / [\text{sum of } (w_i)]$. In averaging a series of measurements with known standard deviations, σ_i , the weight w_i is just $1 / (\sigma_i)^2$. The sigma of a weighted average is $1 / \text{SQRT}(\text{sum of } w_i)$.

Assume all these trials measure the same true rate.

- 1) Calculate the average and Nsd deviation as before
- 2) Calculate the total chi2 and number of degrees of freedom
- 3) Calculate the probability of this chi2

Counts	Duration (min)	Rate (/min)	Sigma	Nsd	Chi2 bin
210	5000	0.042	0.003	-3.0	9.2
280	6000	0.047	0.003	-1.5	2.2
500	10000	0.050	0.002	-0.4	0.1
1100	20000	0.055	0.002	2.5	6.4
20	500	0.040	0.009	-1.2	1.5
49	1000	0.049	0.007	-0.3	0.1
Average					
0.0508	Sigma		Chi2	N d.o.f.	Prob.
	0.0011		19.5	5	0.002

Assume all these trials measure the same true flux.

- 1) Calculate the average flux and Nsd deviation as before
- 2) Calculate the total chi2 and number of degrees of freedom
- 3) Calculate the probability of this chi2

Paddle characteristics									Flux*wt	wt	Chi2 bin
Counts	Duration	Separation	Area 1	Area 2	Flux	Sigma	Nsd				
500	5000	70	400	400	0.510	0.023	0.6	980	1919	0.4	
250	5000	100	400	400	0.521	0.033	0.7	480	922	0.5	
100	5000	150	400	400	0.469	0.047	-0.6	213	455	0.4	
130	10000	200	400	400	0.542	0.048	0.9	240	443	0.9	
160	5000	150	400	650.25	0.461	0.036	-1.0	347	752	0.9	
180	10000	200	400	650.25	0.461	0.034	-1.0	390	846	1.0	
Average											
0.497	Sigma		Chi2	N d.o.f.	Prob.						
	0.014		4.1	5	0.53						

CHI2: The chi2 of a set of measurements X_i to be consistent with an expectation x is $\sum[(X_i - x)^2 / (\sigma_i)^2]$. Here our "expectation" is the average (weighted or other method) of all of all trials. Note that chi2 is always a positive quantity. The number of "degrees of freedom" in a chi2 test is just the number of data points. However, note that if the data itself is used to determine the expectation of x , then this reduces the number of degrees of freedom by one. The chi2 probability is a number between 0 and 1 that indicates how likely it is that for a given number of degrees of freedom, the chi2 could be greater than the reported chi2. A very small chi2 probability indicates that it is unlikely that the measurements X_i are all consistent with the expectation. Hint: the chi2 probability function in Excel is CHIDIST(chi2,ndof)

Now let's attempt to find a linear correlation between count rates and an external variable (in this case, temperature).

- 1) Calculate the average and Nsd deviation, against the "expectation function"
- 2) Calculate the total chi2 and number of degrees of freedom
- 3) Calculate the probability of this chi2
- 4) Vary the parameters of the function (intercept, slope) and see how probability varies
- 5) Find the intercept that provides the minimum chi2 with slope=0. What is the probability?
- 6) Find the intercept that provides the minimum chi2 with slope=0.001. Is the probability significantly higher?
- 7) Repeat for slope=0.002, and slope=0.003

<u>Counts</u>	<u>Duration (min)</u>	<u>Rate (/min)</u>	<u>Sigma</u>	<u>Temp</u>	<u>Prediction</u>	<u>Nsd</u>	<u>Chi2 bin</u>
210	5000	0.042	0.003	68	0.043	-0.3	0.1
280	6000	0.047	0.003	73	0.051	-1.6	2.6
500	10000	0.050	0.002	71	0.048	0.9	0.9
1100	20000	0.055	0.002	75	0.054	0.3	0.1
20	500	0.040	0.009	71	0.048	-0.9	0.8
49	1000	0.049	0.007	70	0.046	0.4	0.2

<u>Intercept</u>	<u>Slope</u>	<u>Chi2</u>	<u>N d.o.f.</u>	<u>Prob.</u>
0.0462	0.0017	4.7	4	0.325
0.0508	0	19.5	5	0.002
0.0476	0.001	7	4	0.138
0.0449	0.002	5.3	4	0.257
0.0433	0.003	14.6	4	0.006

CHI2 Fitting: This is a simple example of "fitting" using a chi2. (More complex examples are possible!) The minimum chi2 gives the highest probability of the prediction explaining the data. Modifying the parameters of the prediction (here, intercept and slope) and watching how the chi2 changes is how a fitter determines the "best values" of parameters meant to describe data. Observation of a much higher probability when introducing a slope parameter as in this example would be evidence for a correlation between the external variable (temperature) and the observed rate.