Chapter 30 Question 5
If you are given a fixed length of wire, how would you shape it to obtain the greatest self-inductance? The least?
Solution
To create the greatest self-inductance, bend the wire into as many loops as possible. To create the least self-inductance, leave the wire as a straight piece of wire.

Chapter 30 Question 8
At the instant the battery is connected into the \( LR \) circuit of Fig. 30-6a, the emf in the inductor has its maximum value even though the current is zero. Explain.
Solution
Although the current is zero at the instant the battery is connected, the rate at which the current is changing is a maximum and therefore the rate of change of flux through the inductor is a maximum. Since, by Faraday's law, the induced emf depends on the rate of change of flux and not the flux itself, the emf in the inductor is a maximum at this instant.

Chapter 30 Question 15
Is it possible for the instantaneous power output of an ac generator connected to an \( LRC \) circuit ever to be negative? Explain.
Solution
Yes. The power output of the generator is \( P = IV \). When either the instantaneous current or the instantaneous voltage in the circuit is negative, and the other variable is positive, the instantaneous power output can be negative. At this time either the inductor or the capacitor is discharging power back to the generator.
Chapter 30 Question 17
Describe briefly how the frequency of the source emf affects the impedance of (a) a pure resistance, (b) a pure capacitance, (c) a pure inductance, (d) an LRC circuit near resonance (R small), (e) an LRC circuit far from resonance (R small).

Solution
(a) The impedance of a pure resistance is unaffected by the frequency of the source emf.
(b) The impedance of a pure capacitance decreases with increasing frequency.
(c) The impedance of a pure inductance increases with increasing frequency.
(d) In an LRC circuit near resonance, small changes in the frequency will cause large changes in the impedance.
(e) For frequencies far above the resonance frequency, the impedance of the LRC circuit is dominated by the inductive reactance and will increase with increasing frequency. For frequencies far below the resonance frequency, the impedance of the LRC circuit is dominated by the capacitive reactance and will decrease with increasing frequency.

Chapter 30 Question 19
An LRC resonant circuit is often called an oscillator circuit. What is it that oscillates?

Solution
In an LRC circuit, the current and the voltage in the circuit both oscillate. The energy stored in the circuit also oscillates and is alternately stored in the magnetic field of the inductor and the electric field of the capacitor.

Chapter 30 Problem 1
A 2.44-m-long coil containing 225 loops is wound on an iron core (average μ=1850μ₀) along with a second coil of 115 loops. The loops of each coil have a radius of 2.00 cm. If the current in the first coil drops uniformly from 12.0 A to zero in 98.0 ms, determine: (a) the mutual inductance M; (b) the emf induced in the second coil.

Chapter 30 Problem 3
A small thin coil with N₁ loops, each of area A₁, is placed inside a long solenoid, near its center. The solenoid has N₂ loops in its length l and has area A₂. Determine the mutual inductance as a function of θ the angle between the plane of the small coil and the axis of the solenoid.
Chapter 30 Problem 9
A coil has $3.25 \Omega$ resistance and 440-mH inductance. If the current is 3.00 A and is increasing at a rate of 3.60A/s what is the potential difference across the coil at this moment?

Chapter 30 Problem 18
Calculate the magnetic and electric energy densities at the surface of a 3.0-mm-diameter copper wire carrying a 15-A current.

Chapter 30 Problem 34
A 425-pF capacitor is charged to 135 V and then quickly connected to a 175-mH inductor. Determine (a) the frequency of oscillation, (b) the peak value of the current, and (c) the maximum energy stored in the magnetic field of the inductor.
For any questions/comments/concerns
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PHY 122, HW #10, Fall '12
Chapter 30 Problem 1

Given

A long coil length L = 2.44 m
with number of turns \( N_1 = 225 \)
wrapped around iron core with \( m = 18.5 \mu \text{m} \)
Second loop has 115 loops = \( N_2 \)
They both have radius \( R = 2.80 \text{ cm} \)

Current in loop 1 drops uniformly from 12.0 A to 0 A
in \( t = 98.0 \text{ ms} \)
\( = 9.8 \times 10^{-3} \text{ s} \)

a) \( M = ? \)

b) \( E_2 = ? \)

Diagram

first wire/loop

second wire
The current carrying first loop will create a magnetic field like it were a solenoid. We are "boosting" the and extending the range of this magnetic field by putting an iron core within the solenoid, filling filling its area and volume. This magnetic field (or rather the changing magnetic flux) will introduce induce a current in the second loop. The proportion of the induced EMF to the changing current per unit time is the mutual inductance constant. This constant is based purely on geometry. Example 30-1 gives a nice derivation of mutual inductance. I quote the result.

\[
M = \frac{\mu_0 N_1 N_2 A}{L} = \mu_0 N_1 N_2 \pi R_a^2 (1850 \mu) = 3.10 \times 10^{-2} \text{ H}
\]

(because they all blue fiys!)}

\[
\varepsilon_2 = -M \frac{dI}{dt} = -M \left( \frac{12.0 A}{98 \times 10^{-3}} \right) = 3.79 \text{ V}
\]
Chapter 30, Problem 3

Given

Thin coil with \( N_2 \) loops and area \( A_2 \) placed within solenoid of \( N_1 \) loops, length \( l \), and area \( A_1 \). 

\[ M = ? \] as a function of \( \theta \).

Diagram

Principle axis of solenoid

Formulation

Once again from the given information, we know the magnetic field strength \( B \). To determine the mutual inductance we need to determine the amount of flux intercepted by the small coil as a function of orientation.
We realize that as $\Theta = 90^\circ$, no magnetic flux is in the direction of the small inner coil and as $\Theta = 180^\circ$, all the magnetic flux is in the direction of the small inner loop. This suggests an inklining that it's a sinusoidal relationship.

We could also arrive at this relationship by considering how much of the magnetic field is in the direction of our coil by doing a vector component de composition.

\[ M = \frac{N_2 \overline{I}_2}{I_1} = \frac{N_2}{I_1} \int \overline{B} \cdot \overline{dA} = \frac{N_2}{I_1} \int B dA \sin \Theta \]

\[ = N_2 \left( \frac{\mu_0 I_1}{2 \pi} \right) A_2 \sin \Theta \]

\[ = \frac{\mu_0 N_2 N_1 A_2}{2 \pi} \sin \Theta \]
Chapter 30 Problem 4

Given: A coil with resistance $R = 3.25 \Omega$ and inductance $L = 440 \times 10^{-3}$ H.

A current $I = 3.00 \ A$ with $\frac{dT}{dt} = 3.60 \ A/s$ is passed through it.

Potential difference $V_{AB} = ?$

Diagram:

\begin{center}
\begin{tikzpicture}
  \node (A) at (0,0) {A};
  \node (I) at (1,0) {I};
  \node (R) at (2,0) {R};
  \node (L) at (3,0) {L};
  \node (B) at (4,0) {B};
  \draw (A) -- (I);
  \draw (I) -- (R);
  \draw (R) -- (L);
  \draw (L) -- (B);
\end{tikzpicture}
\end{center}

Solution:

We can simply write out loop equation

\[ V = IR + LI \frac{dT}{dt} = 11.3V \] and we are done.
Chapter 30, Problem 18

Given

diameter of copper wire \( d = 3.0 \times 10^{-3} \) m

with current \( I = 15 \) A.

\( R \) Magnetic energy density.

\( E \) Electric energy density

\[ R = \frac{B^2}{2 \mu_0} = \frac{\mu_0 I^2}{2 \sqrt{\mu_0 d^2 R^2}} = 1.6 \text{ J/m}^3 \]

\[ E = \frac{E_0 E^2}{2} = \frac{E_0 I^2}{2 \pi \mu_0 d^4} = 5.6 \times 10^{-15} \text{ J/m}^3 \]
Chapter 30 Problem 34

Given

\[ C = 425 \times 10^{-12} \text{ F} \] charged to \[ V = 135 \text{ V} \] then connected to \[ L = 175 \times 10^{-3} \text{ H} \].

a) Frequency of oscillation \( f = ? \)

b) Peak value of current \( I_{\text{max}} = ? \)

c) \( E_{\text{max}} = ? \)

Solution

For formulation

As the capacitor discharges it will create a current through the inductor. The inductor will self-induce a current to oppose this. This current the capacitor will eventually completely discharge and the only current in the circuit is due to the capacitor, inductor. Thus this current it induces in the opposite direction will cause the capacitor to change once again. Thus this cycle repeats.
Solution

a) \( f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi \sqrt{\frac{1}{LC}}} \)

\[ = 18,500 \text{ Hz} \]

b) \( I = wQ_0 \sin (wt + \phi) \) (Eq. 30-15)

\[ \text{max} \ at \ sin (wt + \phi) = 1 \]

\[ I_{\text{max}} = wQ_0 = w_0(U) \]

\[ = 6.65 \text{ mA} \]

c) \( U = \frac{1}{2} U^2 C = 3.8 \text{ mJ} \)