Today in Physics 122: electrostatics review

David Blaine takes the practical portion of his electrostatics midterm (Gawker).
Electrostatics

As you have probably noticed, electrostatics is about calculating forces on electric charges, and work done moving electric charges around, in the presence of fixed (static) distributions of electric charge.

- To the static distribution of charge corresponds an electric field $E(x,y,z)$ and an electric potential $V(x,y,z)$, which can be calculated for any point in space $x,y,z$.

- Usually we calculate the electric field $E$ instead of the force, and obtain the force from $F = qE$.

- And we usually calculate potential difference $\Delta V$ instead of work, and obtain work from $W = q\Delta V$.

- Potential energy is the work done assembling the charges.
The hard way to calculate $E$

… is to use Coulomb’s law, $dE = \hat{r} k dQ/r^2$. It always works, but it involves vector addition and/or vector calculus.

Use when one only has a few point charges, or continuous distributions of charge with no particular symmetry. Steps:

- Determine the number of dimensions of your charge distribution.
- Choose a coordinate system that suits the geometry of the charge distribution and the location at which you need to know the field, and express the charge distribution in that coordinate system.
  - Usually this will still be a Cartesian coordinate system.
If the dimension is zero – point charges, or outside of spherical charges – add the field components of the charges (positions \( r' = x',y',z' \)) at the location in space (position \( r = x,y,z \)) where you need to know the field:

\[
E = \frac{kQ_1}{|r - r'_1|^2} \mathbf{r} - r'_1 + \frac{kQ_2}{|r - r'_2|^2} \mathbf{r} - r'_2 + \ldots
\]

\[
E_x = \frac{kQ_1}{|r - r'_1|^2} \left( \mathbf{r} - r'_1 \right)_x
\]

\[
+ \frac{kQ_2}{|r - r'_2|^2} \left( \mathbf{r} - r'_2 \right)_x + \ldots
\]

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The hard way to calculate \( E \) (continued)
The hard way to calculate $E$ (continued)

- If the dimension is one or more: express the infinitesimal element of charge, and the distance $r - r'$ from the infinitesimal element to the “test” point $r$, in that coordinate system. (2-D shown.)
- Express the **components** $(x,y,z)$ of the unit vector $r - r'$, again in that coordinate system.
- Do the integrals, one vector component at a time: for example,

$$E_x = \int \limits_{\text{bounds of charge distribution}} \frac{k dQ}{|r - r'|^2} (r - r')_x$$
Shortcut to $E$ #1: Gauss’s Law

The flux of $E$ through a closed surface depends only on the charge the surface encloses:

$$\oint E \cdot dA = 4\pi k Q_{encl} = 4\pi k \int_{v} \rho(r') \, dv$$

This is a shortcut to $E$ when

- one is given a distribution of electric charge, and
- the symmetry and/or extent of the charge distribution makes it clear what the pattern (direction!) of $E$ is.
How to use Gauss’s Law to get $E$

- Draw imaginary closed surface (Gaussian surface) through point at which you want to know the field.

- The Gaussian surface needs to be drawn so as to make the unknown $E$ come out of the flux integral: $\int E \cdot dA = |E| \int dA$.

- This usually means the symmetry of surface matches symmetry of the charge distribution.

- Then calculate $Q_{\text{encl.}} = \int \rho dv$ and solve for $E$. 

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Shortcut to $E$ #2: potential

Electric potential is a scalar rather than a vector, and it still obeys the principle of superposition. It is related to the electric field by

$$V(r) = -\int_{\infty}^{r} E \cdot dl \quad \Leftrightarrow \quad E = -\nabla V = -\hat{x} \frac{\partial V}{\partial x} - \hat{y} \frac{\partial V}{\partial y} - \hat{z} \frac{\partial V}{\partial z}$$

- Because it is a scalar, it can be easier to work out than it is to work out $E$ from Coulomb’s law.
- And if you know $V$ and you can take derivatives, you can get $E$. It is, of course, easier to differentiate than to integrate.
How to use potential to get $E$

As long as the charges do not extend to infinity, and as long as you don’t need to know $E$ within a continuous distribution of charges:

- Determine the number of dimensions of the charge distribution.
- Choose a coordinate system that suits the geometry of the charge distribution and the location at which you need to know the field, and express the charge density in that coordinate system. (Usually this will still be a Cartesian coordinate system.)
- Express the infinitesimal element $dQ$, and the distance $|r - r'|$ from the infinitesimal element to the “test” point, in that coordinate system.
How to use potential to get $E$ (continued)

- Then do the integral:

$$V(r) = \int \frac{k\,dQ}{|r - r'|} = k \int \frac{\rho(r') \, dv'}{|r - r'|},$$

- ... and differentiate the result to get $E$:

$$E = -\nabla V = -\hat{x} \frac{\partial V}{\partial x} - \hat{y} \frac{\partial V}{\partial y} - \hat{z} \frac{\partial V}{\partial z}$$
The hard way to calculate $V$

Of course, sometimes one *does* want to know what the potential is, in the two cases

- charge distribution extends to infinity
- $r$ lies within the distribution of charge

which are forbidden for the “Coulomb’s law” version of $V$,

$$V = \int \frac{k dQ}{|r - r'|}.$$  In these cases,

- Derive $E$ first, using Coulomb’s or Gauss’s Law.
  - Most often Gauss’s Law.

- Then integrate the result to get $V$:

$$\Delta V = V(a) - V(b) = -\int_{a}^{b} E \cdot d\ell \quad \text{or} \quad V(r) = -\int_{a}^{\infty} E \cdot d\ell$$
Other nuances of $E$ or $V$ calculations

- **Superposition.** If you are given a distribution of charges that break down into a combination of symmetrical charge distributions: solve the problem for each symmetrical distribution and superpose the solutions, e.g.

  \[ \rho = \rho + (-\rho) \]

- **Conductors.** Perfect conductors are equipotentials, have zero electric field inside, and have surfaces perpendicular to any external electric field.
Quick: how does David Blaine (below) make lightning strike his hands, and how does he live through it?

- He's holding lightning rods in his hands: pointy conductors, which concentrate $E$ at their points because $E$ has to be perpendicular to the conductors.
- He's wearing a conductive suit and helmet.
Capacitance and resistance as examples of $E$ and $V$ derivations

To calculate $C$ or $R$ between two conductors:

- Presume electric charge to be present; say, $Q$ if there is only one conductor, or $\pm Q$ if there are two.

- Either:
  
  - Calculate the electric field from the charges – often using Gauss’s Law – and integrate it to find the potential difference $V$ between the conductors, or
  
  - Solve for the potential difference directly, using $V = \int k dQ / |r - r'|$.

- Then $C_0 = Q / V$.

- If the space between the conductors is filled with dielectric with constant $K$, then $C = KC_0 = KQ / V$. 
Resistivity and resistance (continued)

If the space between the conductors is filled with imperfectly-conducting material with resistivity $\rho$:

- At either electrode, apply the microscopic form of Ohm’s law: $\rho J = E$.
- Multiply through by electrode area $A$; use expressions for $E$ and $V$ to substitute $E$ out, and $V$ in; and use $I = JA$.
- The result will be of the form $V = IR$ (i.e. Ohm’s Law), whence $R = V/I$.
  - This resistor also has capacitance $C_0$, which can be viewed as a $C$ in parallel with its $R$. 

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JE $\rho =$
Capacitors and resistors as circuit elements

Capacitors store charge and energy:

\[ Q = CV \quad u = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C} \]

Resistors carry current and dissipate energy:

\[ IR = V \quad P = I^2 R = \frac{V^2}{R} \]
Combinations of $R_s, C_s$

Parallel and series pairs of resistors and capacitors:

\[
\begin{align*}
\frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} \\
C_{eq} &= \frac{C_1 C_2}{C_1 + C_2}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} \\
R_{eq} &= \frac{R_1 R_2}{R_1 + R_2}
\end{align*}
\]
The midterm exam

Midterm #1: 8-9:15 AM, tomorrow: Friday, 12 October 2012
- Surnames A-K in Hoyt Auditorium
- Surnames L-Z in Lower Strong Auditorium