Today in Physics 122: magnetic force

- Ferromagnetism
- The Biot-Savart law and electromagnetism
- The Lorentz force law
- Calculation of magnetic forces on currents.

Announcements:
- Don’t miss Prof. Demina’s talk on the discovery of the Higgs meson, tomorrow at 4PM, 109 B&L.
- Don’t forget to complete the TA survey.
Ferromagnetism

The first form of magnetism discovered was physically the most complex form: **ferromagnetism**, which gives permanent magnetization to certain metals and alloys.

- Probably found independently in prehistoric Europe, China and India, but first described by the Greeks: earliest mention is Aristotle (late 4th century BC) discussing Thales (early 6th century BC).
- *Ferro-* connotes iron (Fe).
- Magnesia, in Thessaly (central Greece), provided many lodestones (permanently magnetized lumps of iron), and the effect became known as **magnetism** thereby.
  - The region also gave its name to the elements magnesium and manganese.
Ferromagnetism (continued)

Most of the effects are familiar:

- Permanent magnets are dipolar: two magnets will attract each other in one orientation, and repel each other in the other.

- Permanent magnets will attract nonmagnetic samples of certain metals.

- Contact with permanent magnets endows magnetism temporarily to samples of certain materials.
Ferromagnetism (continued)

The poles of a ferromagnet are named north and south, after the direction that a ferromagnetic compass needle points:

- Earth has a dipolar magnetic field, in which the compass needle aligns. The end of the needle pointing toward the north rotational pole – and all ends of magnets repelled by this point – are called North.
Ferromagnetism (continued)

- Magnets cannot be broken into separate poles: break a magnet in two and you get two polarized magnets.
- Magnetic force is communicated through nonmagnetic material.
- The pattern of the forces magnets exert on each other can be illuminated by the orientation of small iron filings sprinkled around in the vicinity of a magnet.
Ferromagnetism (continued)

- The pattern of the lines of force for a permanent magnet resemble in precise detail those of an electric dipole.
- So scientists up to the early 1800s considered ferromagnetism to be due to a field like the electric field, with sources that couldn’t be split into independent “monopoles” like electric charge could.
- ... and thus a phenomenon different from electricity; another force of nature.
Electromagnetism

Then, in the eventful year 1820,

- Biot and Savart (France) discovered that a permanent magnet exerts a force on a current-carrying wire, that goes away when the current is shut off.

- The force they discovered was perpendicular both to the direction of the magnetic field, and to that of the current.
The Biot-Savart force law

- A vector perpendicular to each of two other vectors has the direction of the vector product, or cross product, of those two vectors. In these terms the force discovered by Biot and Savart is

\[ dF = Id\mathbf{\hat{a}} \times B = Id\mathbf{\hat{a}} B \sin \theta \mathbf{\hat{n}} \]

\[ F = \int Id\mathbf{\hat{a}} \times B = \int IB \sin \theta \mathbf{\hat{n}} d\mathbf{\hat{a}} \]

where \( \theta \) is the angle between \( d\ell \) and \( B \), and \( \mathbf{\hat{n}} \) is a unit vector perpendicular to the \( d\ell-B \) plane, in the sense given by the right-hand rule.

Lay fingers of right hand along arc from \( d\ell \) to \( B \), pointing to \( B \). Thumb points along \( dF \).
And Hans Christian Ørsted (Denmark) discovered that an electric current generates its own magnetic field, with circular lines around the current.

Others before him had suspected a deep connection between electricity and magnetism, but this was the first demonstration of the connection. $B$ generated by currents has been called \textbf{electromagnetism} ever since.
The Lorentz force law

A few decades later, people began to get used to the idea of currents as streams of discrete charges in motion, rather than a continuous fluid of charge in flow.

- By the chain rule,

\[ I dl = \frac{dQ}{dt} dl = \frac{dl}{dt} dQ = v dQ \]

so the B-S law implies that the force on a charge \( Q \) moving with velocity \( v \) is

\[ F = Qv \times B \]

as first shown by Maxwell (1865) and Heaviside (1889).

Lay fingers of right hand along arc from \( v \) to \( B \), pointing to \( B \). Thumb points along \( F \).
Units of $B$

The MKS unit of magnetic field is the **tesla**:

$$[B] = \frac{[F]}{[Q][\nu]} = \frac{\text{N s}}{\text{C m}} = \frac{\text{kg}}{\text{C s}} \equiv \text{T (tesla)}$$

different from the units of $E$.

- When physicists do E&M they mostly use CGS units. In this system of units, Coulomb’s Law and the Lorentz Law appear as
  $$F = \frac{Qq}{r^2} \hat{r} = QE \quad \text{No } k$$

  $$F = Q\frac{\nu}{c} \times B \quad c = 2.9979 \times 10^{10} \text{ cm s}^{-1}$$

  $E$ and $B$ have the same dimensions in CGS units. $Q$ and $E$ have different dimensions in MKS than in CGS.
Calculation of magnetic forces

The sideways nature of magnetic forces has vexed students since the beginning of the field, so we should do a few examples.

- A loop of wire, with width L, has two sides bent at 45° from the others, as shown. The pointy end protrudes into a region in which a magnetic field $\mathbf{B}$ exists, perpendicular to the plane of the loop. The wire carries a current $I$. What is the force on the loop?
Calculation of magnetic forces (continued)

- Suppose a length $y$ of the vertical sides protrudes into the field region.
- Note that all the straight wire segments are perpendicular to $B$: that is, $\sin \theta = 1$.
- According to the right-hand rule, the forces on the individual straight segments all point away from the interior of the loop.
So for each segment,

\[ F = \int I dl \times B = \int IB \sin \theta \hat{n} dl \]

\[ = IB \hat{n} \int d\ell = IB \hat{n} \hat{n} \]

where \( \ell \) is the segment’s length and \( \hat{n} \) is perpendicular to the segment, pointing in the plane of the page away from the loop.
Calculation of magnetic forces (continued)

- Break into components in the $x$-$y$ system indicated:

$$F_1 = -IBy\hat{x}$$

$$F_2 = -IB \frac{L\sqrt{2}}{2} \cos 45^\circ \hat{x} + IB \frac{L\sqrt{2}}{2} \sin 45^\circ \hat{y}$$

$$= -\frac{IBL}{2} \hat{x} + \frac{IBL}{2} \hat{y}$$

$$F_3 = +\frac{IBL}{2} \hat{x} + \frac{IBL}{2} \hat{y}$$

$$F_4 = +IBy\hat{x}$$
Calculation of magnetic forces (continued)

- Add them up, and watch all of the $x$ components cancel:

$$F = F_1 + F_2 + F_3 + F_4 = IBL\hat{y}$$

which points vertically upward in the diagram.

- In general for such problems: find a suitable $dl$, find its angle with respect to $B$ considering its location, find $\hat{n}$ or its components, and integrate the B-S law.
The pointy end of the last current loop is replaced by a semicircular wire. Repeat.

As before, the forces on the straight segments are equal and opposite.

Also as before, the semicircle is perpendicular everywhere to the field, so $\sin \theta = 1$ for all $d\ell$.

However, the orientation of $\hat{n}$ is different for $d\ell$ in different locations.
Use polar coordinates just to set up the integral:

\[ d\ell = \frac{L}{2} d\phi, \quad \phi = 0 \rightarrow \pi \]
\[ \hat{n} = \hat{x} \cos \phi + \hat{y} \sin \phi \]

\[ F_{s-c} = \int I d\ell \times B = \int I B \sin \theta \hat{n} d\ell \]

\[ = \frac{IBL}{2} \int_0^\pi (\hat{x} \cos \phi + \hat{y} \sin \phi) d\phi \]

\[ = \frac{IBL}{2} \left[ \hat{x} \sin \phi - \hat{y} \cos \phi \right]_0^\pi \]

\[ = \frac{IBL}{2} \left( -[-1] + 1 \right) \hat{y} = IBL \hat{y} \]
Force on a semicircular current loop (continued)

- That is: the force on the semicircular “point” is the same as that on the 45°-wedge “point”(!?!):

\[ F_{\text{loop}} = IBL\hat{y} \]

- Why?
  - Accident.
  - Same horizontal components of \( I\,dl \).
  - Magnetic forces always point north.
Magnetic forces do no work

Magnetic forces are different in one supremely important respect from all the other forces you know: owing to their sideways nature, magnetic forces can do no work. Observe:

\[
dW = F \cdot ds = (Qv \times B) \cdot ds = Q\left(\frac{ds}{dt} \times B\right) \cdot ds = 0
\]

Perpendicular to \(ds\)

For this reason, one cannot define a scalar potential to go with \(B\), like \(V\) goes with \(E\).

- There is a potential that goes with \(B\), but it turns out to be a vector potential, not as easy to use as \(V\), and thus is beyond the scope of PHY 122.

\[a \cdot b = ab \cos \theta = 0 \text{ if } a \perp b\]