Today in Physics 122: induction

- Ampère’s law applied to a solenoid
- Gauss’s and Ampère’s laws
- Magnetic materials
- Induction and Faraday’s Law
Field in an infinite solenoid, Ampère’s version

Rectangular Ampèrean loop, as shown. The symmetry of the coil dictates that the field must be along $z$, and must be a lot stronger inside than out, so if the number of turns per unit length is $n$, and the current is $I$,

$$\oint B \cdot d\ell = \mu_0 I_{\text{enclosed}}$$

$$B\Delta z = \mu_0 In\Delta z \quad \Rightarrow \quad B = \mu_0 nI\hat{z}$$

Same as before.
Ampère and Gauss

Here are the conditions under which it is profitable to use Ampère’s law to find $B$, compared to Gauss’s law to find $E$.

<table>
<thead>
<tr>
<th>Ampère</th>
<th>Gauss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infinite linear current</td>
<td>Infinite linear charge</td>
</tr>
<tr>
<td>Infinite planar current</td>
<td>Infinite planar charge</td>
</tr>
<tr>
<td>Infinite cylindrical current, any radial dependence</td>
<td>Infinite cylindrical charge, any radial dependence</td>
</tr>
<tr>
<td>Spherically symmetric charge, any radial dependence</td>
<td></td>
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<tr>
<td>Infinite solenoid</td>
<td></td>
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<tr>
<td>Toroid</td>
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</tbody>
</table>

Of course, complicated currents can sometimes be broken into pieces which individually can be treated with Ampère’s law, and then the results superposed.
Magnetic materials

Like dielectrics in electrostatics, there are polarizable magnetic materials which can change magnetic fields.
Magnetic materials (continued)

The magnetic analogy of a parallel-plate capacitor, with its charge $Q$ and uniform $E$ inside, is a solenoid, with its current $I$ and uniform $B$ inside.
Some magnetic materials act like superficially like dielectrics: magnetic dipoles can be induced therein which slightly decrease $B$ for a given $I$. Thus **diamagnetic** material.

$$B = \mu n I < \mu_0 n I$$
Some materials are microscopically analogous to dielectrics: they have embedded permanent magnetic dipoles. But these align so as to **increase** the solenoid field slightly. Such materials are called **paramagnetic**.

\[ B = \mu n I > \mu_0 n I \]
Magnetic materials (continued)

Why do paramagnets act “backwards” compared to dielectric even though they are microscopically analogous?

- Recall which way a current-loop magnetic dipole aligns in an external magnetic field:

\[ \tau = \mu \times B = -IAB \sin \theta \hat{x} \]
Magnetic materials (continued)

...and which way the loop’s own magnetic field and current are oriented, which within their bounds are different from the electric dipole:

$$B(z) = \frac{\mu_0}{2} \frac{IR^2 \hat{z}}{\left(R^2 + z^2\right)^{3/2}}$$

$$B(0) = \frac{\mu_0}{2} \frac{I\hat{z}}{R}$$
But diamagnetic and paramagnetic materials change the magnetic field by typically 0.0001%-0.001%. **Ferromagnetic** materials have very large permeability $\mu$:

\[ B = \mu nI \gg \mu_0 nI \]

\[ K_m = \frac{\mu}{\mu_0} \sim 10^3 \]
Magnetic materials (continued)

Furthermore, ferromagnets aligned in solenoids act like permanent magnets, and can polarize other ferromagnetic materials in the neighborhood.

\[ B = \mu nI \]
Like the flux of $E$ – considering which, we came up with Gauss’s Law – it is useful to consider the flux of $B$:

$$
\Phi_B = \int B \cdot dA
$$

Loop of area $A$:

- **$\perp$** to $B$: $\Phi_B = BA$
- Inclined by $\theta$ from $B$: $\Phi_B = BA \cos \theta$
Flux of $B$ (continued)

Ferromagnets can be used to “conduct” magnetic flux around (see two pages back).

- Here, the flux in the loops on the left and right, both wrapped around a ferromagnetic torus, are the same.
Michael Faraday was experimenting with this setup in 1831, and found that when he changed the current in the left-hand circuit, he produced a transient (short-lived) current in the right-hand one.
He noticed, by use of a galvanometer on the right-hand loop, that the direction this transient current flows is that which produces a magnetic field, and change in magnetic flux, in the opposite direction from that produced by the change in the current in the left-hand loop.
Since electromotive forces are what drive currents, Faraday summarized his measurements with this relation for the EMF in the right-hand circuit:

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

the minus sign representing the “opposition.”
The minus sign itself is usually referred to as **Lenz’s Law**, after the physicist who showed that the sign had to be negative to be consistent with conservation of energy. Faraday was already used to thinking of an EMF that drives a current in a circuit enclosing the flux as corresponding to an electric field that moves electric charge, so he furthermore wrote the former in terms of the latter:

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

\[ \oint_C E \cdot d\ell = -\frac{d}{dt} \int_S B \cdot dA \]

*Faraday’s Law*
Example induced EMF and current

Wire loop aligned perpendicular to $B$: Turn the magnetic field off abruptly. Which direction does this cause current to flow?

- Flux into the page decreases. Thus current will flow in the direction that increases magnetic flux in this direction, which in this case is clockwise.
Example induced EMF and current

After the current dies off, turn the magnetic field back on abruptly. Which way does current flow?

- Counterclockwise, which now is the direction which opposes the change: it produces its own magnetic field in the direction opposite that of the applied field.
Example: moving wire

A conducting circuit perpendicular to a uniform $B$ has one side moving perpendicular to its length and $B$, at speed $v$. Calculate the induced EMF.

- The rate of flux change is
  \[
  \frac{d\Phi_B}{dt} = B \frac{dA}{dt} = BLv = -\mathcal{E}
  \]

The minus sign indicates the direction that generates $B$ out of the page: counterclockwise.