

Name:

Partners' Names:

Laboratory Section:

Laboratory Section Date:

Grade:

Last Revised on December 5, 2005

EXPERIMENT 4

Moment of Inertia & Oscillations

1. Purpose

In the first part of this laboratory exercise you will measure the moment of inertia of three different objects about a specified rotational axis and verify the parallel axis theorem. In the second part you will measure the oscillations of a mass on a spring to investigate Hooke's Law.

2. Introduction

2.1 Moment of Inertia and Parallel-Axis Theorem

Consider a rigid body rotating about an axis, as in *Figure 4.1*. If the angular velocity is ω , each point in the body will move with linear speed $r\omega$ where r is the perpendicular distance of the point from the rotational axis. The total angular momentum L of the rotating body points along the axis and is equal in magnitude to,

$$L = \int_{\text{Body}} r v dm = \int_{\text{Body}} r^2 \omega dm = \omega \int_{\text{Body}} r^2 dm = I\omega \quad \text{Equation 4.1}$$

where

$$I = \int r^2 dm \quad \text{Equation 4.2}$$

is called the *moment of inertia* of the body about the axis of rotation. In the MKS system of units, the unit of I is $\text{kg} \cdot \text{m}^2$. If the axis of rotation is chosen to be through the center of mass of the object, then the moment of inertia about the center of mass axis is called I_{CM} . For example, $I_{CM} = \frac{1}{2}MR^2$ for a solid disk of mass M and radius R (see *Figure 4.1*). *Table 4.1* gives examples of I_{CM} for some objects with different mass distributions.

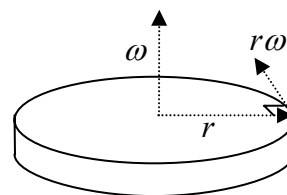


Figure 4.1

The *parallel-axis theorem* relates the moment of inertia about an axis through the center of mass I_{CM} to the moment of inertia I about a parallel axis through some other point. The theorem states that,

$$I = I_{CM} + Md^2 \quad \text{Equation 4.3}$$

where M is the total mass of the body and d is the perpendicular distance between the two axes. This implies I_{CM} is always less than I about any other parallel axis.

Object	Rotational Axis	I_{CM}
Thin Ring	Symmetry Axis	MR^2
Thick Ring	Symmetry Axis	$\frac{1}{2}M(R_1^2 + R_2^2)$
Solid Disk	Symmetry Axis	$\frac{1}{2}MR^2$
Thin Spherical Shell	About a Diameter	$\frac{2}{3}MR^2$
Solid Sphere	About a Diameter	$\frac{2}{5}MR^2$

Table 4.1

When working with rotational motion for rigid bodies, many of the equations are similar to the equations of motion for linear motion. The angular velocity is used instead of linear velocity, and the moment of inertia is used instead of the mass. *Table 4.2* summarizes the correspondence between linear and rotational kinematics for rigid bodies.

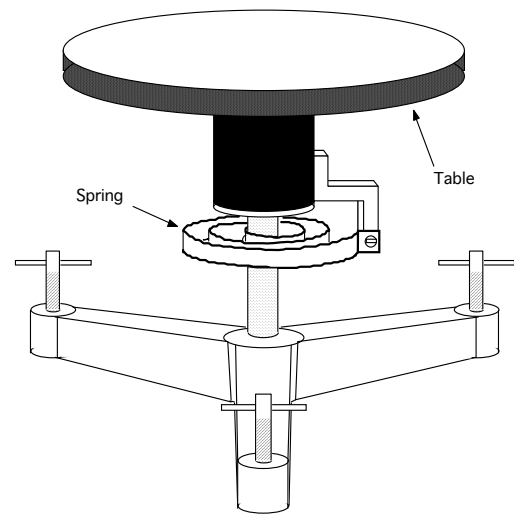


Figure 4.2

Linear Kinematics	Rotational Kinematics
Velocity = v	Angular Velocity = ω
Mass = M	Moment of Inertia = I
Momentum = $p =$ Mv	Angular Momentum $m = L =$ $I\omega$
Kinetic Energy = $K =$ $\frac{1}{2}Mv^2$	Kinetic Energy = $K =$ $\frac{1}{2}I\omega^2$

Table 4.2

The apparatus in this experiment in *Figure 4.2* consists of a rotary table on which you can mount an object in order to measure its moment of inertia. A torsion spring restricts the motion of the table and provides a restoring torque. If the table is rotated by an angle Φ the torque acting on it will be equal to,

$$\tau = -K\Phi \quad \text{Equation 4.4}$$

where K is the “force constant” for a torsional spring, which must be measured. If the sum of the moment of inertia of the table I_{Table} and that of the mounted object I is equal to $I + I_{Table}$, the table will perform a rotary oscillation with the frequency,

$$\omega = \sqrt{\frac{K}{I + I_{Table}}}, \quad \text{Equation 4.5}$$

which corresponds to a period of oscillation,

$$T = 2\pi\sqrt{\frac{I + I_{Table}}{K}}, \quad \text{Equation 4.6}$$

Note that there are two unknown parameters of the apparatus: I_{Table} and K . To determine these parameters, you need to measure the period T_{Table} of the table alone and the period $T_{Table+Object}$ of the table with an object of a known moment of inertia I_0 placed on the table. From *Equation 4.6*, one finds that

$$T_{Table} = 2\pi\sqrt{\frac{I_{Table}}{K}}, \quad \text{Equation 4.7}$$

and

$$T_{Table+Object} = 2\pi \sqrt{\frac{I_0 + I_{Table}}{K}} . \quad \text{Equation 4.8}$$

Using the technique of solving sets of equations, you square both equations and add them to obtain,

$$(T_{Table+Object})^2 + (T_{Table})^2 = 4\pi^2 \frac{2I_{Table} + I_0}{K} , \quad \text{Equation 4.9}$$

and by squaring them and subtracting one from the other you get,

$$(T_{Table+Object})^2 - (T_{Table})^2 = 4\pi^2 \frac{I_0}{K} . \quad \text{Equation 4.10}$$

This allows you to solve for one of the unknown constants,

$$K = 4\pi^2 \frac{I_0}{(T_{Table+Object})^2 - (T_{Table})^2} . \quad \text{Equation 4.11}$$

The other unknown I_{Table} can be obtained first by dividing *Equation 4.9* by *Equation 4.10* to get the following,

$$\frac{(T_{Table+Object})^2 + (T_{Table})^2}{(T_{Table+Object})^2 - (T_{Table})^2} = \frac{2I_{Table}}{I_0} + 1 , \quad \text{Equation 4.12}$$

and then simplifying and solving for I_{Table} ,

$$I_{Table} = I_0 \frac{(T_{Table})^2}{(T_{Table+Object})^2 - (T_{Table})^2} , \quad \text{Equation 4.13}$$

By these means the unknown constants of the table, I_{Table} and K , can be determined if a body of known moment of inertia is available. Now knowing the values of I_{Table} and K , you can find the moment of inertia I_x of an unknown object on the table by using the equation below,

$$I_x = I_{Table} \left(\left(\frac{T_{Table+x}}{T_{Table}} \right)^2 - 1 \right) , \quad \text{Equation 4.14}$$

where $T_{Table+x}$ is the measured period of the table with the unknown object.

2.2 Hooke's Law and Spiral Spring Oscillations

It is often assumed that a long spiral spring obeys Hooke's law if it is not stretched too far. If the spring is hung vertically from a fixed support and a mass is attached to its free end, the mass can then oscillate vertically in a simple harmonic motion pattern by stretching and releasing it. The period of an oscillation depends upon the attached mass M and the spring force constant k , assuming the mass of the spring m is negligible. The time it takes for the weight to return once to the starting position is defined as one period. If the mass of the spring m is negligible, then the period T is,

$$T = 2\pi \sqrt{\frac{M}{k}} . \quad \text{Equation 4.15}$$

However, since the spring itself also moves and its mass is not negligible, the spring mass

must enter into the equation for the period. Since the equation must remain dimensionally correct, we correct for the mass in the following way:

$$T = 2\pi\sqrt{\frac{M + bm}{k}}. \quad \text{Equation 4.16}$$

where b , the *spring mass coefficient*, is a dimensionless constant to be determined. In this experiment its theoretical value is $1/3$.

3. Laboratory Work

3.1 Moment of Inertia and Parallel-Axis Theorem

Procedure

1. After the table reaches its equilibrium position, make sure that the table top is level. Place a level on it and adjust the table apparatus' feet if necessary.
2. Place the timer's photogate on one side of the table's trigger. Set the timer's mode to pendulum. (See *Figure 4.3*.)
3. Wind the table in such a direction that *the trigger does not pass the timer while winding*. Stop at a point before the timer. So the amount of winding should be *less than one*.
4. Release the table. Count the number of times the trigger passes through the photogate before the timer stops counting. It should be three passes before the timer stops and the time on the timer represents the period of one oscillation. Repeat winding and releasing the table to make sure your setup is done correctly and timer is working properly.
5. Measure the mass, inner and outer radii of the *brass ring*. Record these values in the *Section 4.1*.
6. Measure the period of one oscillation T_{Table} of the *table alone* five (5) times. Record the data in *Table 4.3*.
7. After making sure the brass ring is mounted down with screws and centered on the table's axis of rotation, measure the period of one oscillation $T_{Table+Ring}$ of the *table/ring combination* five (5) times. Record the data in *Table 4.3*.
8. Measure the mass and radius of the solid disk. Record the data in *Section 4.1*.
9. To measure the period of one oscillation for the table/disk combination, place and screw down the solid disk at one of five different equally spaced positions along the table radius at a distance d from the table's axis of rotation (see *Figure 4.4*). One of the positions must be the center ($d = 0.000\text{m}$). Recommended positions are 0.000m , 0.015m , 0.030m , 0.045m and 0.060m .
10. Measure the period of one oscillation at five different disk positions. Measure each period five times. Record your data in *Table 4.4*.

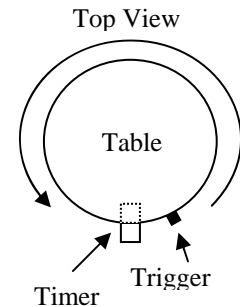


Figure 4.3

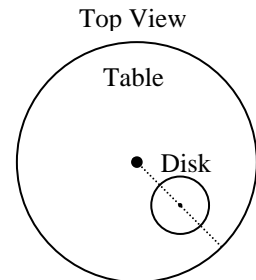


Figure 4.4

3.2 Hooke's Law and Spiral Spring Oscillations

Introduction

According to Hooke's law, the extension of a spring should be proportional to the force exerted by the spring on an attached mass— $F = -kx$, where F is the force exerted by the spring, x the extension of the spring, and k the *force constant* of the spring. You will test the validity of Hooke's law by making measurements of spring extension as increasingly greater amounts of mass are attached. Then, you will measure the oscillation period of several different hanging masses to measure the spring mass coefficient needed to take into account the spring's own mass m .

Procedure

1. Weigh the spring to determine its mass. Record it in Section 4.2.
2. Mount the spring by clipping one end of the spring to the flat metal piece mounted to the stand.
3. Let the spring come to equilibrium.. Record the position of the spring's bottom end. This is the spring's equilibrium length.
4. Measure the change in the extension of the spring x as you attach different amounts of mass to the end of the spring. Keep in mind that the "extension of the spring" is the length of spring extended *beyond the spring's equilibrium length*. Use masses of 5g, 10g, 15g, 20g and 25g. Attach the masses gently. Record the data on *Table 4.6*.
5. Measure the period of *ten* oscillations for the same five different attached masses. Set the spring and mass into motion by *stretching* the mass gently by about 15cm to 20cm and then releasing it. Be careful not to destroy the spring by over-stretching it. While in motion, the mass should not touch the floor or anything else. Have the same person who releases the spring also starts and stops the timer. Divide the period by 10 to get the average period of *one* oscillation. Record the one-oscillation periods on *Table 4.6*.

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Moment of Inertia & Oscillations

4. Post-Laboratory Work [18pts]

4.1 Moment of Inertia and Parallel-Axis Theorem [9pts]

Mass of Ring (kg) = _____
 Ring's Inner Radius (m) = _____
 Ring's Outer Radius (m) = _____

Trial	T_{Table}	$T_{Table+Ring}$
1		
2		
3		
4		
5		
Average		

Mass of Solid Disk (kg) = _____
 Radius of Solid Disk (m) = _____

Table 4.3

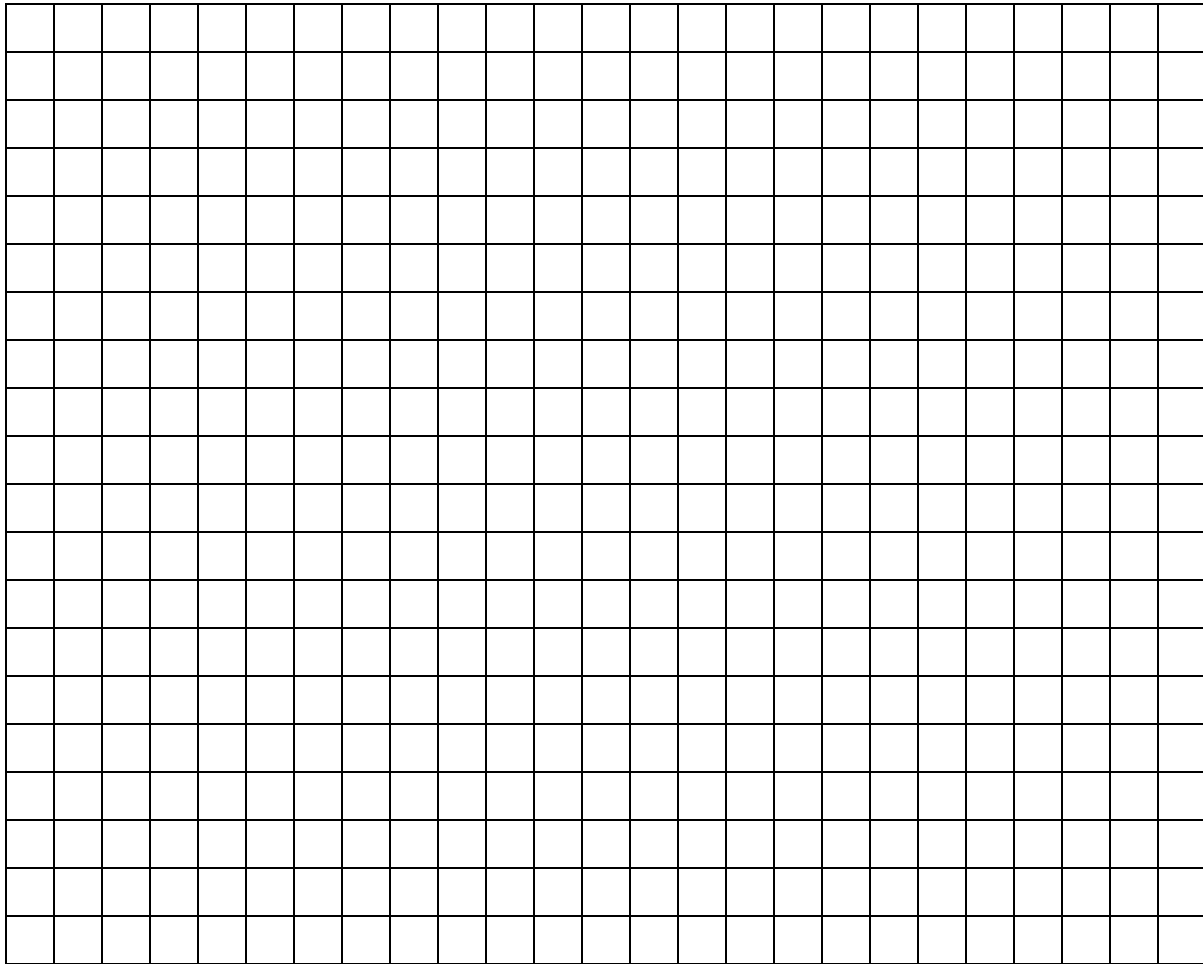
Distance (m)	$d_{x_1} =$ _____	$d_{x_2} =$ _____	$d_{x_3} =$ _____	$d_{x_4} =$ _____	$d_{x_5} =$ _____
Trial	T_{x_1}	T_{x_2}	T_{x_3}	T_{x_4}	T_{x_5}
1					
2					
3					
4					
5					
Average (sec)					

Table 4.4

$I_{Ring} =$	Theoretical Value of the Moment of Inertia of the Brass Ring (<i>Table 4.1</i>)
$K =$	Spring Constant of the Table (<i>Equation 4.11</i> , where $I_0 = I_{Ring}$)
$I_{Table} =$	Calculated Moment of Inertia of the Table (<i>Equation 4.13</i> , where $I_0 = I_{Ring}$)
$I_{Disk} =$	Theoretical Value of the Moment of Inertia of the Brass Disk (<i>Table 4.1</i>)
$I_{x_1} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_1 (<i>Equation 4.14</i> , where $T_{Table+x} = T_{x_1}$)
$I_{x_2} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_2
$I_{x_3} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_3
$I_{x_4} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_4
$I_{x_5} =$	Calculated Moment of Inertia when the Brass Disk is placed at d_5
Error = $\left I_{Disk} - I_{x_1} \right =$	Calculated Error in Moment of Inertia of the Solid Disk

Table 4.5

1. Calculate all the quantities in *Table 4.5*. As for the values of the measured periods, use the average values from *Tables 4.3* and *4.4*. Write the formula for each quantity except from I_{x_2} to I_{x_5} and the Error. Include units in the calculated values. You do not have to show calculation work. [3pts]
2. On the graph below, plot the moment of inertia of the *solid disk* versus the *square* of the distance between the centers of the table and center of the solid disk, i.e. I_{x_x} vs. $d_{x_x}^2$. Remember that, as on all graphs, plots are y vs. x and you should title the graph, label the axes and include units. Also include error bars on each data point for the moment of inertia (y -axis) using the Error from *Table 4.5*. [2pts]



3. Draw a best-fit straight line through the error bars of the data points. As always with drawing best-fit lines, do not simply connect lower-end and upper-end data points. Also do not force it to go through the origin. Find the y-intercept value and slope of the best-fit line. Circle the two points on the best-fit line that were used in the slope calculation. Also circle the y-intercept point on the graph (not necessarily a data point). Include units in your calculation results. [1pt]

4. If you are to plot I vs. d^2 (as done in **Question 2**) of the formula for the Parallel-Axis Theorem, $I = I_{CM} + Md^2$, what do you expect the y-intercept and slope values to be according to this Theorem? [1pt]

5. Compare the theoretical values of the y-intercept and slope as found in **Question 4** with those you calculated in **Question 3** based on your measurement. For both y-intercept and slope, by how much does the measured value differ from the theoretical value? [1pt]

6. Name two most likely sources of experimental error. Give a reason as to how each source of error would affect your values for the moment of inertia. [1pt]

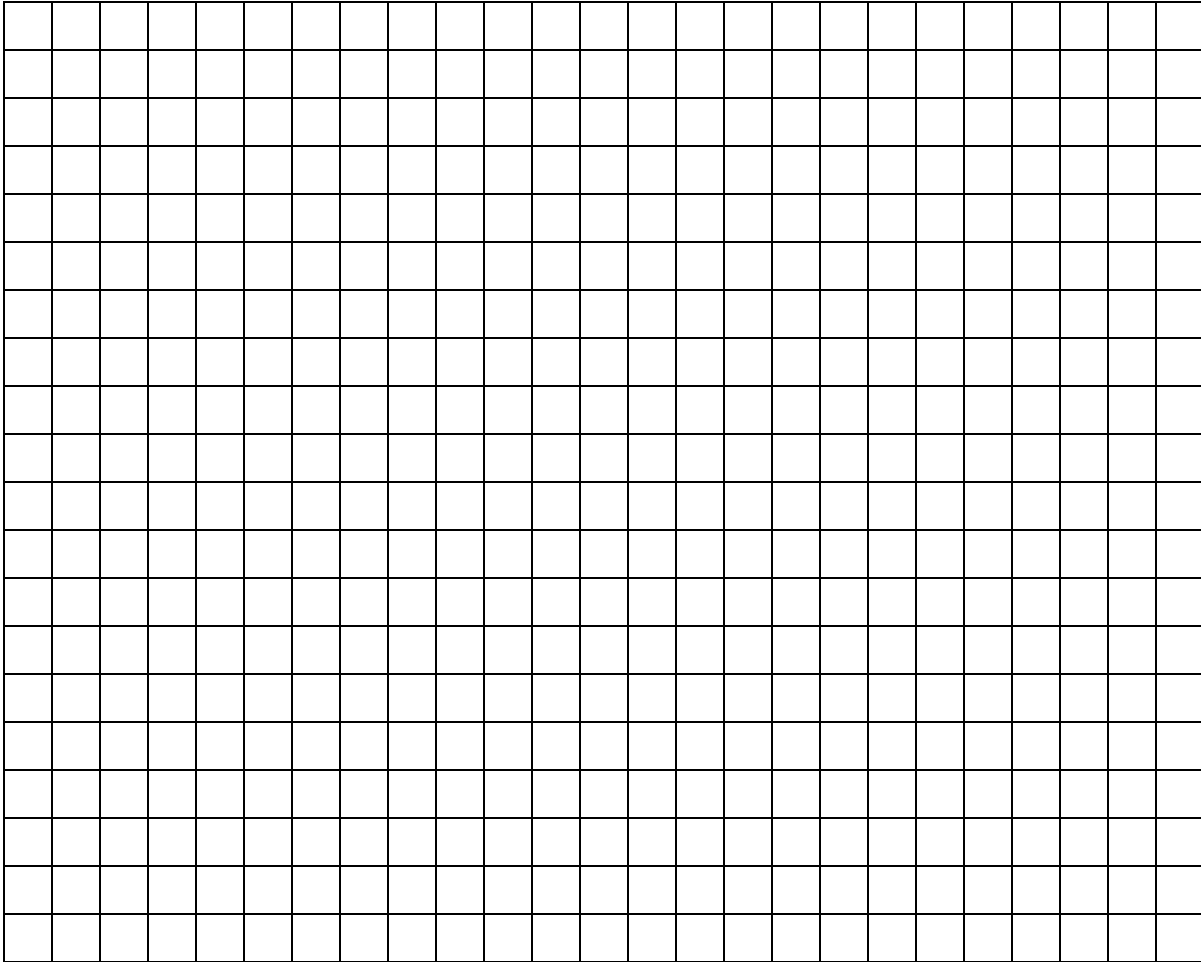
4.2 Hooke's Law and Spiral Spring Oscillations [9pts]

Mass of Spring (kg) = _____

Attached Mass (kg)	Force of Mass (N)	Extension (m)	Period (sec)	Period Squared (sec ²)
0.005				
0.010				
0.015				
0.020				
0.025				

Table 4.6

7. Plot Extension versus Force of Attached Mass (y vs. x) using the data from *Table 4.6*. Include title, axis labels and units. Draw a best-fit straight line through the data points. [1pt]

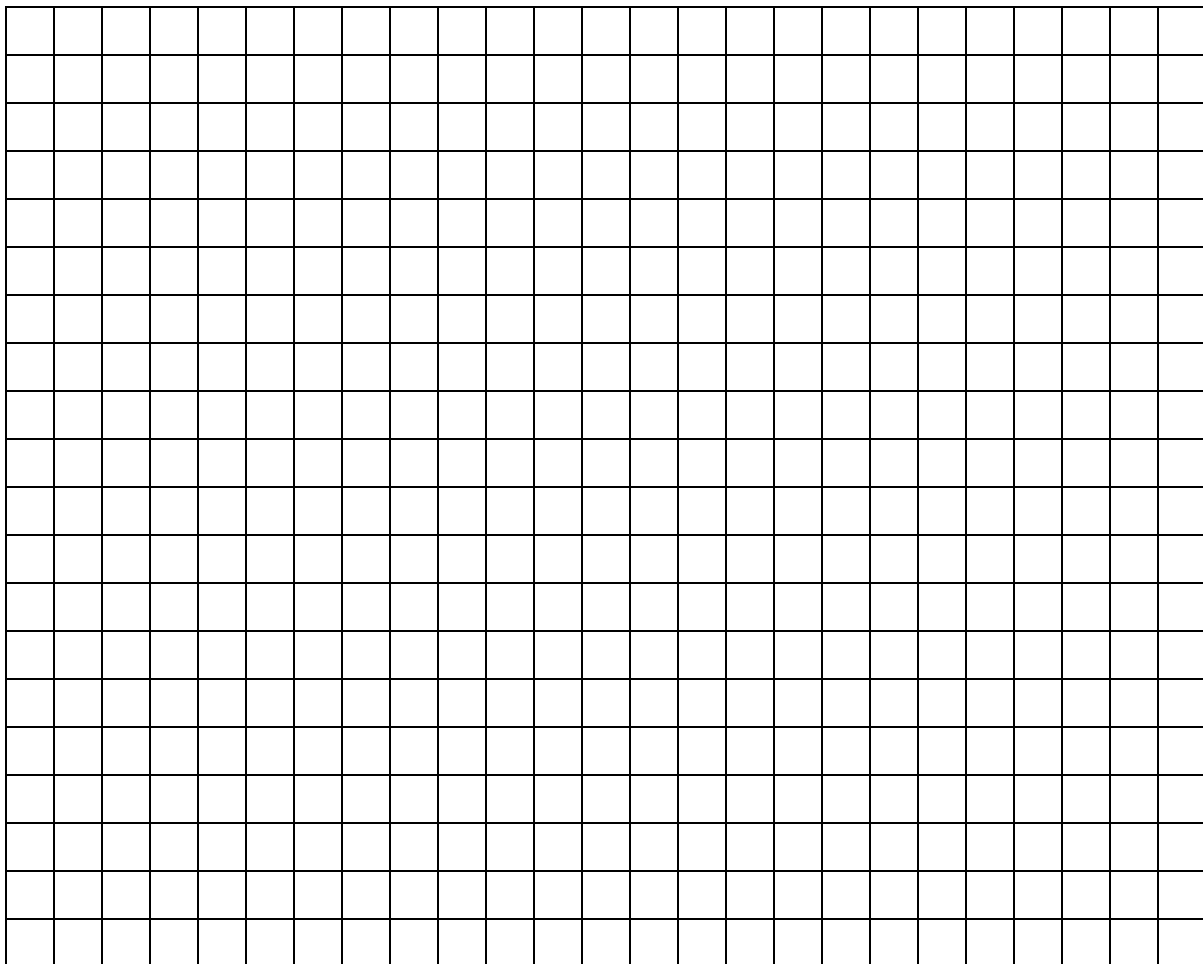


8. Based on the best-fit straight line, do your measurements verify the Hooke's Law? Briefly explain. [1pt]

9. As done in **Question 3**, calculate the slope of the best-fit straight line. Circle the two points on the line that were used in the calculation. Find the force constant of the spring (spring constant) by taking the inverse of the slope. Include the unit. [1pt]

10. What would be the term (not a numerical value) for the y-intercept for a plot of T^2 vs. M according to *Equation 4.16*? What would be the term for the slope? Start by squaring the both sides of the equation. [1pt]

11. Plot the period squared versus the attached mass, i.e. T^2 vs. M (y vs. x) using the data from *Table 4.6*. Draw a best-fit straight line. Include title, axis labels and units. [1pt]



12. Calculate the slope of the best-fit straight line. Circle the two points on the line used in the calculation. Knowing the numerical value of the slope and the slope term found in **Question 10**, find the force constant of the spring (spring constant) k . [1pt]

13. Find the y-intercept value of the best-fit straight line. Similar to **Question 12**, find the spring mass coefficient b by plugging the numeric value of the y-intercept to the y-intercept term found in **Question 10**. Use the k value calculated in **Question 12**. [1pt]

14. Note that *Equation 4.16* becomes *Equation 4.15* if bm is negligible compared to M . Suppose we can say bm is negligible compared to M if $\frac{bm}{M} \leq 0.25$. Based on your calculated spring mass coefficient b , can you say the mass of spring can be neglected in this experiment? [1pt]

15. Compare the force constants of spring, k , from **Questions 9** and **12**. In **Question 9** the constant was calculated based on the Hooke's Law, whereas in **Question 12** it was calculated based on a simple harmonic motion. Comparing to the lesser of the two, by how much do they differ? Are they comparable? Suppose they are comparable if the difference is equal to or less than 10% of the lesser of the two. [1pt]