Name: *Partners' Names:*
Laboratory Section:
Laboratory Section Date: *Grade:*

Last Revised on September 26, 2006

EXPERIMENT 6

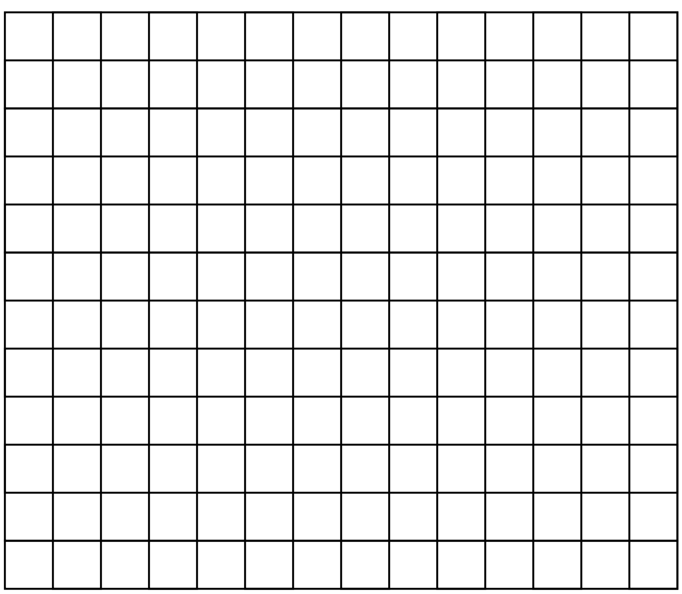
Coulomb's Law

0. Pre-lab Homework [2 pts]

The pre-lab homework must be handed to the lab TA at the start of the lab.

1. What is the purpose of measuring the rate at which charge leaks off of the metal spheres?(1pt)

2. Consider the function $y = x^n$ (for n = -2). Take the natural logarithm of both sides and plot ln(y) vs. ln(x) on the grid below. Explain or show how to obtain "n" from the graph. (Hint: Can you fit the graph below to a straight line?) (1pt)



ln(x)	ln(y)

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Coulomb's Law

1. Purpose

The purpose is to verify the proportionality of Coulomb's Law, that is, to verify that the electric force between two point charges is directly proportional to the product of the charges and is indirectly proportional to the square of the distance between them.

2. Introduction

Coulomb's Law gives us the static electrical force F, exerted by a point charge Q_1 on another point charge Q_2 in terms of r, the distance between them:

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2}$$ **Equation 6.1**

In this experiment, we will verify this law and also learn how to use an optical lever to magnify a small rotation into a large displacement. Furthermore, we will learn about systematic errors and how to correct for them.

3. Laboratory Work

3.1 Procedural Outline

The verification of Coulomb's Law proceeds as follows. A diagram can be found in Figure 6.1.

1. Charges are placed on the spherical conductors with a charged Lucite rod.
2. Due to the presence of the charge, a force is induced between the two spheres.
3. The force (F) between the spheres will produce a deflection (ϕ) of the sphere attached to the torsion fiber. These two quantities are related in the following way,

$$F = k\phi \quad \textbf{Equation 6.2}$$

where k is the *torsion constant* of the fiber.

4. The charge dependence is measured by removing charge (to Q_1, for instance), keeping the spheres' separation constant, and measuring the resulting change in ϕ (thus inferring F).
5. The separation (r) dependence is measured by charging the spheres and measuring the resulting change in ϕ (thus inferring F) for various separations, r.

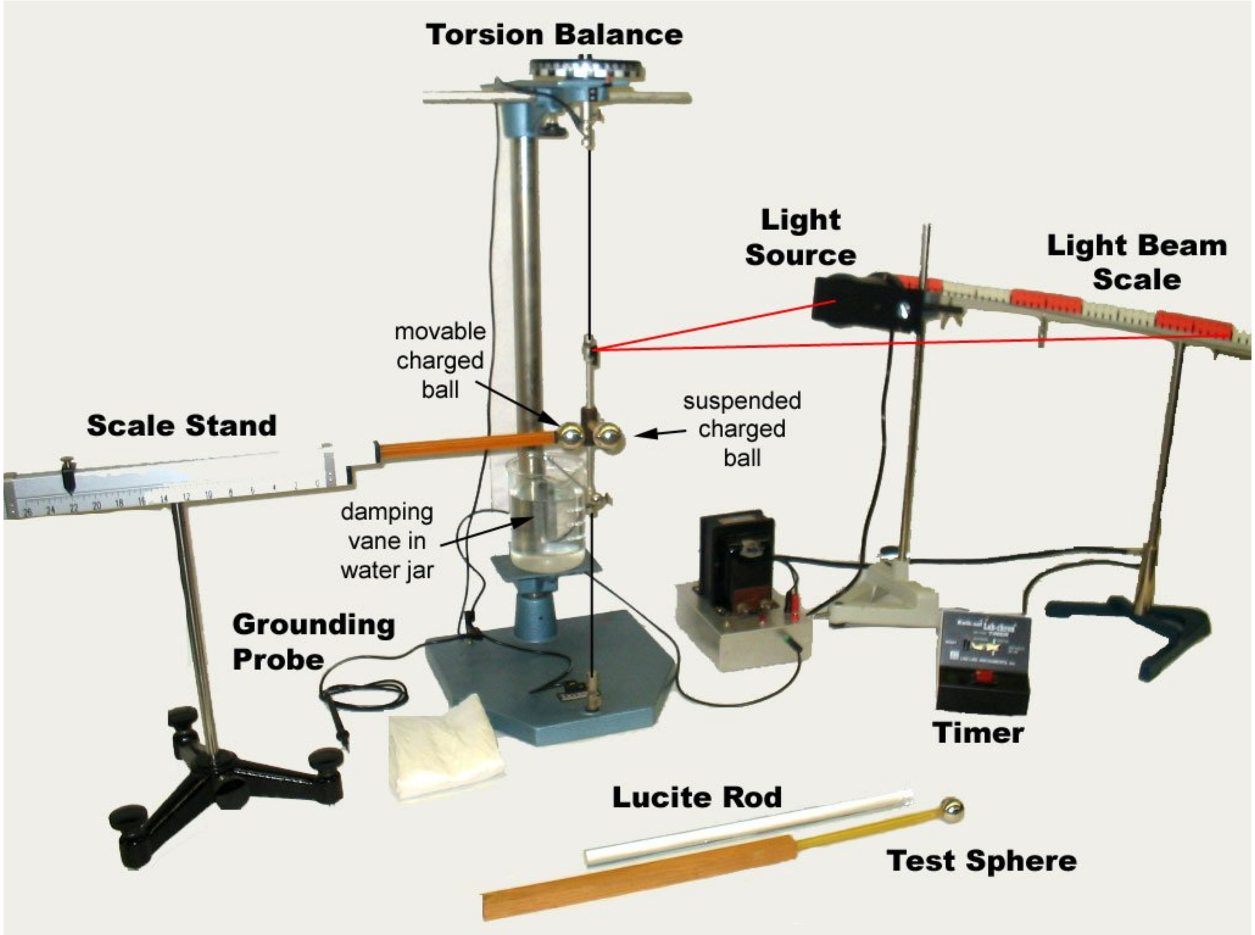

Figure 6.1

3.2 Setting Up the Apparatus

1. Clean the spheres and insulators with alcohol; dry them with tissue paper and dryer. This ensures that no charges apart from the ones we place on the spheres influence the experiment.

2. The optical lever system for measuring must be adjusted. Adjust the torsion head so that the wire is taut and the damping vane is at the center of the beaker so that it can move without hitting the glass walls of the beaker.

3. Adjust the lamp and place the stand of the scale 1.2m away from the mirror to get a sharp image about 25cm from the right end (looking from the mirror) of the circular scale. Each mark on the scale is 1 cm. Make sure that the image does not go off the scale at the

maximum deflection (ϕ_o) of the light beam. The light source, the mirror and the scale should all be in one horizontal plane.

4. Remember, the deflection (ϕ) is a **displacement** quantity: you measure deflection by observing the **change** in the light beam's position at equilibrium compared to some new position.

3.3 Measuring the relaxation time of the torsion fiber

Although we are interested in the static deflection of the optical lever system to infer the force *F*, we have to acknowledge that this system is a torsion oscillator and the transient behavior must be determined. There is an oscillatory period t_o and a damping or relaxation time t_r. The angular displacement due to these transients is given by:

$$\phi(t) = \phi_o \exp\left(-t/t_r\right)\cos\left(2\pi t/t_o\right) \qquad \textbf{Equation 6.3}$$

from which we can see that at time $t = t_r$

$$\phi(t) = \phi_o \exp(-1) \approx \phi_o/3 \qquad \textbf{Equation 6.4}$$

(Here we have used the fact that for critical damping, $t_o \sim t_r$.) The reason we wait until the deflection becomes one third of the original ϕ_o is that then $t = t_r$ which is a measure of the time it takes the torsion oscillator to settle down. Each time we vary the charge or the distance and need to take a measurement of deflection, we should wait approximately $3t_r$ seconds before taking down a reading to let the torsion oscillator reach its equilibrium value.

1. Touch the spheres with the grounding probe - this makes them neutral or grounded. The grounding probe is the small probe attached by a wire to ground on the torsion balance.

2. Perturb the optical lever system by blowing on the sphere attached to the torsion. Obtain about a 15-20cm deflection (ϕ_o). Note this maximum deflection and start the timer. Stop when deflection, ϕ, is about ϕ_o /3. This time measured is the relaxation time, t_r. Record t_r. in *Section 4.1.*

3.4 Measuring the leakage of charge

Because of air currents and humidity, charge tends to leak away from the spheres. The leakage is not negligible and has to be accounted for. In this section you will measure the leakage of charge with time. This will enable you to make a correction to the charge in your later measurements.

1. Adjust the movable sphere such that the pointer on the scale reads 3.0 cm; bring the movable sphere and stand as near as you can to the sphere on the torsion without making them touch. Since the radius of each sphere is 1.5cm the distance between their centers (3cm) can now be

directly read off the scale. The heights of the spheres from the table should be equal. The stand should not be moved for the rest of the experiment.

2. Rub the Lucite rod with silk. Draw the Lucite rod across the movable sphere (like drawing a bow across the strings of a violin – ask your TA to demonstrate) to transfer its charge.

   This will introduce an opposite "image" charge on the sphere attached to the fiber. It will move towards the movable sphere attached to the scale, finally touching it. Once the spheres touch, the charges will be equally distributed between them and since they now will have the *same* charge, they will begin to *repel* each other. The sphere on the torsion fiber will now begin to move away from the movable sphere, and the light image will register a deflection on the circular scale. You should keep increasing the charge until $\phi = \phi_o$ = 25cm (This is not critical; 20 or 32 cm will work just as well). Cleaning the Lucite rod with alcohol and air-drying by waving in the air will ensure you have sufficient charge to transfer. Start the timer.

   An Important Note on Timing. You will be making measurements of deflection, ϕ, very often in this lab. Each time you measure ϕ, you will also note the time and record them as pairs. Essentially, you are putting a "time-stamp" on each data point. You will do this for each set of measurements so that you can keep track of how much charge 'leaks' off of the spheres, because you will know exactly when each measurement was taken.

3. As time elapses, ϕ will decrease. You will record ϕ vs. time (t). For every centimeter of change in ϕ (i.e. 25, 24, 23cm…) record the elapsed time. Record at least 10 data points in Table 6.1 of *Section 4.2*. The leakage of charge may turn out to be an exponential decay, i.e. the deflection at any time t would be given by

$$\phi(t) = \phi_0 \exp(-t/t_d) \qquad \textbf{Equation 6.5}$$

   where ϕ_o is the initial deflection and the decay is characterized by a "time constant", t_d.

4. Once you have completed Table 6.1 of *Section 4.2*, ask your TA or TI to check your data. This will help you identify any problems and verify that not only is your setup correct, but that environmental conditions did not affect your results. If anything is wrong your TA and TI may adjust your apparatus or assist you in carrying out the procedure again so that you can obtain better results.

3.5 Varying the charge Q_1

Now we will actually begin making measurements to test Coulomb's Law. In this section we vary the charge.

1. Ground the spheres with the grounding probe and obtain equilibrium.
2. Set movable sphere to 3.5 cm. Charge the spheres to get a 20-30cm deflection.

3. Reset movable sphere to 5cm. Start the timer. Note the deflection ϕ_o and time (t) after the transient dies out. (Do **not** stop the timer.)

   Note : If we set r = 5cm *before* charging, the amount of charge we will have to put on the spheres to get a 20-30cm deflection is going to be rather large - remember that the electric force F (proportional to ϕ according to Equation 6.2) is proportional to $1/r^2$, i.e., it weakens rapidly with distance.

4. Ground the test sphere, the metal sphere on a wooden handle; touch it to the movable sphere, thus halving its charge. Record ϕ, and time t after the transient dies out. Here we used the fact that the uncharged test sphere, when brought into contact with the movable sphere (carrying charge Q_1) will take away half its charge, leaving it with $Q_1/2$. To vary Q_1 further, we will *ground* the test charge and touch it to $Q_1/2$, halving it again to $Q_1/4$ and then record the corresponding ϕ, and time t in Table 6.3 of *Section 4.3*.

5. Repeat the halving procedure until you reach $Q_1/8$ or the deflection is too small to measure.

3.6 Varying the separation *r* between the spheres

The procedure is similar to that of *Section 3.5* above.

1. Ground the spheres with the ground probe and obtain equilibrium.
2. Set the movable sphere to 3.5 cm. Charge spheres to achieve 20 - 30 cm of deflection.
3. Reset the movable sphere to 5 cm. Reset and start the timer. Measure and record deflection ϕ_o and time t after the transients have died down in Table 6.4. Do **not** stop the timer.
4. Set the sphere on the scale to 10, 15, and 20 cm; record ϕ, and time t after each change of separation in the table in *Section 4.4*. Note that we do not change the charge amounts in this part of the experiment. The charge will only change due to leakage, which we will account for later.

3.7 [OPTIONAL – Ask your TA] Leakage of charge (again)

Due to possible changes in humidity and other conditions of the air we measure the decay curve and plot it on the same graph as *Section 3.4*. Record the data in the Table 6.2 of *Section 4.2*.

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The post-laboratory work must be completed and handed to the lab TA at the end of lab.

4. Post-laboratory Report [18 pts]

4.1 Torsion Pendulum Relaxation Rate [1pt]

Torsional Relaxation Rate, (t_r): ________________ (1 pt)

4.2 Charge Leakage from *Section 3.4 & 3.7* [3 pts]

1. Record your data from charge leakage measurements in the tables below. (1 pt)

Table 6.1 (Section 3.4)

Time	ϕ	$\ln(\phi_o / \phi)$
t=0	ϕ_o =	

Table 6.2 (Section 3.7)

Time	ϕ	$\ln(\phi_o / \phi)$
t=0	ϕ_o =	

2. Plot the normalized deflection vs time decay curve {ln[ϕ_o / ϕ] vs. time} (y vs. x !!) on the grid provided below for your charge leakage data set(s) from Table 6.1 (and Table 6.2). Be sure to label the axes. Provide a key if necessary. If the deflection is exponential, you should observe a straight line. Draw a "best fit" line for each data set. (1pts)

3. Extract the slope of each line. (Compute an average slope if you have 2 data sets). Show the calculation below. Then determine t_d, knowing that the slope of the line is a measure of the charge leakage time constant as discussed in Equation 6.5, such that: $\textbf{slope (m)} = {}^{\mathbf{1}}\!/_{\mathbf{t_d}}$ We will be using this value and Equation 6.5 to make *decay corrections* to the data taken in *Sections 3.5 & 3.6*. (1 pt)

4.3 Analysis of Force vs. Charge [7 pts]

4. Record the data from *Section 3.5* in Table 6.3. Then correct for charge leakage to obtain a set of data that is *decay-corrected*. The data point at time zero will not need to be corrected. To correct for the data, take the measured value for displacement, say $\phi_{measured}$ (for $q = Q/2$) that has a time stamp, $t_{1.}$ and then, from Equation 6.5 and knowing the value of t_d , we can use the relation:

$$\phi_{\text{corrected}} = \phi_{\text{measured}} e^{\left(t_1 / t_d \right)} \qquad \textbf{Equation 6.6}$$

Table 6.3

	Time	ϕ_{measured}	$\phi_{\text{corrected}}$
Q			
1/2 Q			
1/4 Q			
1/8 Q			

Show at least one example calculation below of how you *decay-corrected* your data. If you want to show all the calculations, you may do so. (2 pt)

5. Plot your corrected data for deflection versus the charge Q ($\phi_{corrected}$ vs. Q, y *vs.* x) from Table 6.3 on the grid below. Provide axes labels. Draw a best-fit curve (i.e. a line, smooth curve, etc.) to your data. (2 pts)

6. Analyze your best-fit curve to the deflection vs. charge plot. What mathematical relationship exists (approximately) between the deflection and charge? (1pt)

7. By recognizing the relationship set forth in Equation 6.2, what should the theoretical relationship be between the decay-corrected deflection and the charge according to Coulomb's Law? Why or why not does your data agree with Coulomb's law? (2pt)

4.4 Analysis of Force vs. Separation [7 pts]

8. Perform a similar analysis of the separation data from *Section 3.6* as you did for the charge data from *Section 3.5*. You will need to *decay-correct* each deflection value, ϕ, and record it in Table 6.4. Give at least one example of such a calculation in the space provided beneath the table. The decay correction equation, Equation 6.6 is provided again: (2 pts)

$$\phi_{\text{corrected}} = \phi_{\text{measured}} e^{\left(t/t_d \right)}$$

Table 6.4

r	Time	ϕ_{measured}	$\phi_{\text{corrected}}$	$\ln(\phi_{\text{corrected}})$
5cm				
10cm				
15cm				
20cm				
25cm				

9. Make a plot of ln($\phi_{corrected}$) vs. ln (r) (y vs. x !!) from Table 6.4 on the grid below. Make a best-fit curve to the plotted data. You may wish to 'weight' lightly the data for small separations (5, 10cm) as image charge effects are the largest there and the graph might deviate from the expected behavior. Include axes labels. (2 pts)

10. What is the slope of the best-fit line? Show your calculation below. (1pt)

11. Do the results of the graph and slope indicate a "power law" ($y = x^n$)? If so, what is your best estimate of the exponent? How does your result compare with Coulomb's Law? Hint: Consider what you did in Pre-lab Question #2. (2 pts)