**PHY 408 Quantum Mechanics II**

Final Exam Spring 99

One and a Half hours

1. Consider a particle moving in the plane under the influence of a potential $V(x) = \frac{1}{2}\lambda x^2 y^2$ for positive $\lambda$.

(i) Show that in the classical limit, there are trajectories in which the particle can escape out to infinity with an arbitrarily small energy.

(ii) Show that in the quantum theory, all states are bound states.

**Hint** Show that $H = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] + \frac{1}{2}\lambda x^2 y^2 \geq -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}\hbar\sqrt{\left[\frac{\lambda}{m}\right]}|x|$ and hence that even the ground state is a bound state.

2. The third component $J_3$ of the angular momentum of a particle of total angular momentum $j = \frac{1}{2}\hbar$ is measured and found to have a value $-\frac{1}{2}\hbar$ . Then the the component $J_1 \sin\theta + J_3 \cos\theta$ of the angular momentum is measured. (Here, $0 \leq \theta \leq 2\pi$ is an angle). What are the possible outcomes? What is the probability of each outcome, as a function of $\theta$ ?.

3. Consider a particle moving in one dimension with the hamiltonian, (in units where $\hbar = 1$), $H = -\frac{d^2}{dx^2} + V(x)$. Let $E_n$ be the energy of the $n$th energy level.

(i) Let $V(x) = \lambda x^{2k}$, where $\lambda > 0$ and $k$ a positive integer. Determine the asymptotic dependence of $E_n$ on $n$, as $n \to \infty$. Verify that your answer is correct for the special case $k = 1$ .

(ii) Find a potential such that, as $n \to \infty$, $E_n \sim n^2$.