PHY 408 Quantum Mechanics II

Midterm Exam S99

1. Consider a particle moving in one dimension with the hamiltonian

$$H = -\frac{d^2}{dx^2} + g|x|^k$$

with $k > 0, g > 0$. Find how the energy of the n th energy level depends on g and n as $n \to \infty$.

2. A particle of mass m moves in an ellipsoidal potential well:

$$V(x,y,z) = 0 \text{ if } x^2 + y^2 + \frac{z^2}{c^2} < 1$$

and $V(x,y,z) = \infty$ otherwise. Find the ground state energy of the particle to first order in the difference $c - 1$.

Hint Change variables so that the boundary is spherical in shape, and write the hamiltonian in the new variables as $H = H_0 + gH_1$ where H_0 is the free hamiltonian and gH_1 is a small correction.

3. A particle of spin half moves on the surface of a sphere with the hamiltonian

$$H = \mu\sigma \cdot \mathbf{L}.$$

Here, σ are the Pauli matrices and $\mathbf{L}$ the orbital angular momentum operators. Find the eigenvalues of the hamiltonian and their degeneracies.