PHY 408 Quantum Mechanics II

Problem set 1 Spring 1999

1. A linear operator B is *positive* if $< u, Bu > \geq 0$ for all u with equality only for $u = 0$. For given any operator A , show that $AA^\dagger$ and $A^\dagger A$ are positive operators.

2. Show that any hermitean 2×2 matrix A can be written as a linear combination of Pauli matrices and the identity: $A = a_0 1 + a_1\sigma_1 + a_2\sigma_2 + a_3\sigma_3$. What are the eigenvalues and eigenvectors of A?

3. Consider the linear operator $\mathcal{F} : L^2(R) \to L^2(R)$, (the Fourier transform)

$$\mathcal{F}\psi(p) = \int \psi(x) e^{-ipx} \frac{dx}{\sqrt{(2\pi)}}.$$

(i)Show that $\mathcal{F}^2\psi(x) = \psi(-x)$ and hence that $\mathcal{F}^4 = 1$.

(ii)What are the eigenvalues of $\mathcal{F}$?. Find at least one example of an eigenvector and its corresponding eigenvalue.

(iii)* Find *all* the eigenvectors corresponding to each eigenvalue; i.e., find an orthonormal basis in each eigenspace.

The problems marked with a * are harder and are meant as challenges to advanced students.