PHY 408 Quantum Mechanics II

Problem set 2

1. Let $\mathbf{L} = \mathbf{r} \times (-i\nabla)$ represent the orbital angular momentum operators.

(i) Express them in terms of the angular variables. Show that they do not involve radial co-ordinate.

(ii) Find the simultaneous eigenfunctions of L^2 and L_3, in the special case where the eigenvalue of L_3 is zero.(You will get an ordinary differential equation which has a solution you can look up in some textbook.)

(iii) Find all the simultaneous eigenvectors of L^2 and L_3.

2. The hamltonian of a rigid rotor is,

$$H = \frac{L_1^2}{2I_1} + \frac{L_2^2}{2I_2} + \frac{L_3^2}{2I_3}$$

where L_1, L_2, L_3 satisfy the angular momentum algebra and I_1, I_2, I_3 are positive numbers.

(i) When $I_1 = I_2 = I_3$ what is the spectrum of eigenvalues of H?

(ii) When a pair of the eigenvalues are equal, say, $I_1 = I_2$ what is the spectrum?

(iii) [1] Show that $L^2 = L_1^2 + L_2^2 + L_3^2$ commutes with H even in the generic case where no pair of the numbers I_1, I_2, I_3 are equal. Find the exact eigenvalues of H for as many values of L^2 as you can.

3. An atom with total angular momentum $j = \frac{3}{2}$ passes through a magnetic field which prepares it in an eigenstate of L_3 with eigenvalue $-\frac{1}{2}$. If we now measure L_1, what are the possible outcomes? What is the probability of each outcome?

[1] This is an open ended problem which has no complete solution. I want to see how far you can go with it, especially the advanced students.