PHY 408 Quantum Mechanics II S99

Problem set 3

1. Consider the Hamiltonian

$$H = A\sigma_3 + \lambda \mathbf{a} \cdot \sigma$$

where $\mathbf{a}$ is a unit vector, and $|\lambda| << |A|$. The components of σ are the Pauli matrices.

(i) . Find the exact eigenvalues of this hamiltonian.

(ii) . Find the eigenvalues to the first two orders of perturbation theory. Compare with the exact answer.

(iii) . What is the radius of convergence of the expansion for the eigenvalues as a power series in λ ?. What is the nature of the singularity of the eigenvalues when viewed as functions on the complex λ -plane?

2. A hermitean $n \times n$ matrix is detremined by n^2 parameters: the vector space of such matrices is n^2 dimensional. How many independent conditions should these parameters satisfy in order that one pair of eigenvalues coincide? (In more precise mathematical language, what is the co-dimension of the subspace of matrices with one pair of coincident eigenvalues?) **Hint** Consider first the case of 2×2 matrices. The general case is meant for those who consider themselves advanced students.

3. Consider the anharmonic oscillator

$$H = H_0 + gx^4$$

where $H_0 = \frac{1}{2}[p^2 + \omega^2 x^2]$. Treat g as a small parameter. Find the energy eigenvalues to second order and eigenvectors to first order of perturbation theory.

4. Consider the hamiltonian for the rigid body

$$H = \frac{L_1^2}{2I_1} + \frac{L_2^2}{2I_2} + \frac{L_3^2}{2I_3}$$

where I_1 and I_2 are approximately equal. Here, L_1, L_2, L_3 are angular momentum operators and I_1, I_2, I_3 the moments of inertia along the principal directions. Find the energy eigenvalues upto second order of perturbation theory in the parameter $g = \frac{1}{2}\left(I_1^{-1} - I_2^{-1}\right)$.