PHY 408 Quantum Mechanics II S99

Problem set 4

1. A uniform electric field is suddenly applied to a harmonic oscillator which changes its hamiltonian from

$$H_0 = \frac{1}{2}[p^2 + \omega^2 x^2]$$

to

$$H = H_0 + gx.$$

What are the eigenstates of H ? Assuming the oscillator was in the ground state of H_0 initially, what is the probability that it will be found in the n th excited state of H ?

2. Consider the hydrogen atom immersed in an electromagnetic plane wave. In the dipole approximation, the perturbation due to such a plane wave is

$$H_1(t) = -e\mathbf{x} \cdot \mathbf{E} e^{i\omega t}.$$

$-e\mathbf{x}$ is the dipole moment of the atom and $\mathbf{E}$ the electric field amplitude. Find the transition amplitude from the ground state to a first excited state.

3. Let H_0 be the hamiltonian of an electron moving in the (x, y) plane, in the presence of a constant magnetic field B normal to the plane: the ground state energy is infinitely degenerate. Now suppose there is a perturbation

$$gH_1 = g\delta(x)\delta(y) \text{ for } g < 0.$$

The Dirac delta function represents a deep but narrow potential well located at the origin. Find the ground state energy and wavefunction to first order of degenerate perturbation theory.

Hint This is harder than the other problems but everyone should be able to do it. If you can solve this case study the case of two such wells

$$gH_1 = g[\delta^2(\mathbf{x} - \mathbf{a}) + \delta(\mathbf{x} + \mathbf{a})]$$