PHY 408 Quantum Mechanics II Spring 99

Problem set 5

1. Consider the anharmonic oscillator

$$H = \frac{1}{2}\left[\frac{-d^2}{dx^2} + x^2\right] + \lambda x^4$$

with $\lambda > 0$.

(i) . Estimate the ground state energy using the Gaussian as a trial wavefunction. Verify that in the limit $\lambda = 0$ we get the exact answer.

(ii) . Compare the variational estimate with first order perturbation theory. Which should be a better approximation?

2. Find the ground state energy of the anharmonic oscillator numerically and compare with the above variational approximation.

Hint

Express hamiltonian of the anharmonic oscilltor above in the basis of the eigenstates $|n>$ of the harmonic oscillator:

$$H_{nn'} = (n + \frac{1}{2})\delta_{nn'} + \lambda < n|x^4|n' > .$$

You can get a formula for this by expressing x in terms of the creation–annihilation operators. For small values of λ you can approximate this infinite dimensional matrix by a finite sub-matrix of the first r columns and rows. You can diagonalize this finite matrix for each value of λ using any of several standard programs. For example, Mathematica has built in functions that diagonalize matrices. There are standard packages such as "Numerical Recipes" in C or Fortran. The value of r will have to grow with λ but you can start with $r = 10$ and test if the answer changes if $r = 15$, say. Mathematica is easy to use and is avaliable on crunch.pas.rochester.edu. Please learn about unix so that you can use the computer without interfering with others.

3. Determine the scattering matrix, as a function of the wavenumber k , for a delta-function potential in one dimension:

$$H = -\frac{d^2}{dx^2} + \lambda\delta(x).$$

Determine the singularities of the scattering matrix in the complex k plane. What is the physical meaning of the poles?

4. Determine the s -wave partial scattering cross-section $\sigma_0(ka)$ for a spherical well:

$$V(r) = V_0 \quad \text{for} \quad r < a$$

and $V(r) = 0$ otherwise. Plot $\sigma_0(ka)$ as a function of ka . Here k the wavenumber.