

PHY254 Nuclear and Particle Physics Fall 2007 S. G. Rajeev

Problem Set 1 due Wed Sep 17 2008

1 An alpha particle of mass m is scattered by a nucleus of finite mass M . Let Θ be the scattering angle of the alpha particle in the reference frame where the nucleus is initially at rest. Recall that this problem can be transformed into that of a single particle of reduced mass $\mu = \frac{mM}{m+M}$ scattered through some angle θ by a center that does not recoil. Find a relation between Θ and θ by using conservation of energy and momentum.

2 Consider the scattering of an alpha particle of mass m by an electron whose mass m_e is much smaller: $m_e \ll m$. Show that there is a maximum possible scattering angle and show that it is very small. This justifies why we can ignore the electrons in Rutherford Scattering.

3 What is the minimum energy (in MeV) that an alpha-particle must have in order that it sees the effect of a finite nuclear size for Gold? The radius of the Gold nucleus is about 8.15 fm. Assume that outside of the nucleus, the alpha-particle only feels the electrostatic force.

Problem Set 2 due Wed Oct 1st 2008

4 . Suppose there are N_0 nuclei of half-life T in a sample.

4.1 How many decays do you expect in some time t , if $t \ll T$?

4.2 If this number is λ , what is the probability that there will be r decays, for $r = 0, 1, 2, \dots$?

5 . The unit of radioactive decays, a Curie, is defined to be 3.7×10^{10} decays per second. This is roughly the number of decays of one gram of Radium²²⁶.

5.1 Knowing that an atomic mass unit is 1.66×10^{-27} kilograms, estimate how many Radium atoms there are in one gram.

5.2 Find the half-life of Radium²²⁶.

5.3 What is the probability that the number of decays of a nanogram of radium in a tenth of a second is r ? Plot these probabilities as a function of r for $r = 0, 1, 2, \dots$

6 . Consider a particle of mass m that moves on the half-line $x > 0$ under the potential

$$V(x) = \begin{cases} 0 & \text{for } 0 < x < x_0 \\ V_0 \frac{x_1 - x}{x_1 - x_0} & \text{for } x_0 < x < x_1 \\ 0 & \text{for } x > x_1 \end{cases} \quad (1)$$

for some constant $V_0 > 0$. What is the probability that the particle of energy $0 < E < V_0$ will tunnel to the outside, once it arrives at x_0 ?

Problem Set 3 due Wed Oct 22nd 2008

7 Recall that orbital angular momentum is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and $\mathbf{p} = -i\hbar\nabla$.

7.1 Find the commutation relation of the different components of angular momentum.

7.2 Show that $L^2 = L_1^2 + L_2^2 + L_3^2$ commutes with all the components of \mathbf{L} .

7.3 What are the eigenvalues of L^2 ? For a given eigenvalue of L^2 , what are the eigenvalues of L_3 ?

7.4 What are the matrices representing angular momentum for orbital quantum number $l = 0, 1, 2, 3$?

You may consult a book on Quantum Mechanics. But show details of proofs, except for part 7.3.

8 Find matrices satisfying angular momentum commutation relations for spin $\frac{1}{2}$.

8.1 If we combine two spin half particles, what are the allowed values of spin of the combined system?

8.2 Which value of spin occurs for the symmetric combination of states? Which for anti-symmetric combination?

9 The energy of a system is given by

$$H = \frac{L_1^2}{2I_1} + \frac{L_2^2}{2I_2} + \frac{L_3^2}{2I_3} \quad (2)$$

for some constants I_1, I_2, I_3 .

9.1 When $I_1 = I_2$ find the eigenvalues of H .

9.2 If $I_1 = I_2 > I_3$ what is the lowest eigenvalue of H ? Same for $I_1 = I_2 < I_3$.

Problem Set 4 due Wed Nov 5th 2008

10 The mass of a deuteron is 2.01355 u and the masses of a proton and neutron are 1.00728 u and 1.00867 u respectively. Find the binding energy per nucleon of the deuteron. Is it justified to use non-relativistic quantum mechanics in understanding the binding of the deuteron? 3/10 points

10.1 Model the interaction between the neutron and proton by a potential well

$$V(r) = -V_0 \quad \text{for } r < a, \quad V(r) = 0 \quad \text{for } r > a. \quad (3)$$

for two positive constants V_0 and a . Using the known binding energy, find V_0 assuming a range of $a = 2$ fm. Make reasonable approximations, as this model is approximate anyway. 7/10 points

11 The neutron and the proton are the two states of a nucleon with isospin $I = \frac{1}{2}$. What are the possible values of isospin and spin for an np boundstate? Take into account the fact the nucleon is a fermion and assume that in the ground state the orbital angular momentum is zero. 3/10 points

11.1 It is known experimentally that the deuteron has spin one. What does this tell us about the isospin of the deuteron? 3/10 points

11.2 The proton has a magnetic moment of 2.8 nuclear magnetons¹ and the deuteron a magnetic moment of 0.85 in the same units. What is the magnetic moment of the neutron? What is its direction in relation to its spin? 3/10 points

12 It is known from Dirac's theory that elementary particles of charge e , mass m and spin $\frac{1}{2}$ should have a magnetic moment equal to twice $\frac{e\hbar}{2m}$. The electron's magnetic moment is understood this way, but not that of the proton or the neutron. This leads to the idea that they are made of more elementary particles, the quarks. Assume that the quarks are spin half particles with isospin $\frac{1}{2}$ as well. The isospin up state is the up quark and the other is the down quark. Then

¹A nuclear magneton is $\frac{e\hbar}{2m_N}$ where m_N is the mass of the nucleon; i.e., the average mass of the neutron and the proton.

the simplest possibility is that the proton and the neutron are made of three quarks. (Why not two?)

12.1 What should be the electric charges of the quarks? Which combinations (quantum mechanical states) make up the neutron and the proton?

12.2 Assuming that the average mass of the up and down quarks is roughly a third of the mass of the nucleon, calculate the magnetic moments of the neutron and the proton. How close is the prediction to the actual values?

12.3 What other three-quark states are allowed? Make a table of them, with their electric charges and (if you can) predict their magnetic moments.