

Syllabus for PHY 401 Complex Analysis and Differential Equations
S. G. Rajeev Fall 01

There is no required textbook. Useful references are *Applied Complex Variables* by Dettman, *Theory of Functions of a Complex Variable* by Copson, *Complex Analysis* by Ahlfors and *Advanced Mathematical Methods for Scientists and Engineers* by Bender and Orszag.

Homeworks will be handed out roughly every week. About 50% of the grade will be based on the homeworks; There will be a midterm exam worth 20% and a final exam worth 30% of the grade.

1. The field of complex numbers; norm of a complex number; the complex plane; stereographic map of the sphere.
2. Closed and open sets; convergent sequences; infinite series; absolute convergence; double series. Infinite products.
3. Definition of a continuous function of a complex variable; Definition of an analytic function; Cauchy–Riemann equations; harmonic functions.
4. Polynomials; Power series; exponential and related functions.
5. Contour integrals; Cauchy’s theorem; Taylor’s theorem; Laurent’s theorem. Residue calculus; Principal value; Hilbert transform.
7. Entire functions; Product representation of trigonometric functions. Weierstrass functions. The Gamma function. Product formula. Asymptotic formula.
8. (Time Permitting) Divergent series. Borel summation; Pade approximants.
1. Linear ordinary differential equations; ordinary point, regular and irregular singular points; the point at infinity; Local behaviour near regular singular points.
2. Solution near an ordinary point; wronskian; solution near a regular singular point;
3. The hypergeometric equation. Solution by power series. Integral representation. Behaviour at infinity.
4. Bessel functions; Legendre functions; Hermite’s equation. Sturm–Liouville problems.
5. (Time Permitting) Local behaviour near irregular singular points; asymptotic series.
6. Fourier and Laplace transforms; solution of differential equations by Fourier and Laplace transforms.
7. Linear second order partial differential equations; canonical form; hyperbolic, elliptic and parabolic types; Cauchy problem for the wave equation.