S10Symmetries
Problem Set 2
Due Feb 17 2010

(1) Recall that $su(3)$ is the Lie algebra of anti-hermitean $3 \times 3$ matrices. What is its dimension? (i.e., the number of independent basis elements). What is the rank of this Lie algebra? (i.e., the maximum number of commuting basis elements) Find a basis for $su(3)$. (Hint: Gell-Mann matrices play a role here analogous to Pauli matrices for spin.) Find a positive quadratic form that is invariant, analogous to $L^2$ for $so(3)$.

(2) For a given magnitude of angular momentum $L$, and principal moments of inertia $A_1 < A_2 < A_3$, what is the maximum rotational kinetic energy allowed for a rigid body? What is the minimum energy?

(3) Solve the Euler equations in terms of the Jacobi functions using the ansatz

$$L_1 = C_1 \cn(\omega t, k), \quad L_2 = C_2 \sn(\omega t, k), \quad L_3 = C_3 \dn(\omega t, k)$$

Determine how $C_1, C_2, C_3, \omega, k$ depend on $K, A_1, A_2, A_3, L^2$. 