S10Symmetries
Problem Set 3
Due Mar 3 2010

Exercise 1. This problem is an open ended challenge. Define $P_j(z_1, z_2, z_3) = \det\left[z_1 L_1^2 + z_2 L_2^2 + z_3 L_3^2\right]$ where L_1, L_2, L_3 form an irreducible representation of dimension $2j+1$(i.e., spin j) .Calculate and solve this polynomial (i.e., write as a product of lower order polynomials and solve each one) for as large a value of j as you can.**Hints** Recall that polynomials of order ≤ 4 are solvable in terms of radicals. For $j = \frac{1}{2}, 1, \frac{3}{2}$ you should be able to do this by hand. Use all the symmetries and algebraic tricks you can think of. For example, even and odd values of L_3 are not mixed in the matrix. It will help to use a symbolic algebra program like Mathematica or Maple for larger values of j.

Exercise 2. Show that the symmetric combinations of a pair of spin $\frac{1}{2}$ states $|\uparrow\uparrow\rangle \quad \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}, \quad |\downarrow\downarrow\rangle$ are eigenstates of $(\mathbf{J} + \mathbf{J}')^2$ with eigenvalue 2; and that the antisymmetric combination has $(\mathbf{J} + \mathbf{J}')^2$ equal to zero.

Exercise 3. Show that a completely symmetric tensor $\psi^{A_1 A_2 \cdots A_r}$ of order r , where each index can take n values, has $\frac{n(n+1)\cdots(n+r-1)}{r!}$ independent components. Similarly a completely antisymmetric tensor has $\frac{n(n-1)\cdots(n-r+1)}{r!}$ independent components. In particular, how many states of baryon number two can you form using a pair of nucleons? How many states of baryons in the static quark model (assuming antisymmetry under color)?