

S10Symmetries

Problem Set 5

Due April 21 2010

- (1) Show that the commutator of covariant derivatives is just a multiplication; derive a formula for the field strength in a non-abelian Yang-Mills theory:

$$\nabla_\mu \phi = \partial_\mu \phi + iA_\mu \phi$$

$$\nabla_\mu \nabla_\nu \phi - \nabla_\nu \nabla_\mu \phi = iF_{\mu\nu} \phi$$

- (2) Show that under gauge transformations,

$$A_\mu \rightarrow gA_\mu g^{-1} + g\partial_\mu(g^{-1})$$

$$F_{\mu\nu} \rightarrow gF_{\mu\nu}g^{-1}.$$

- (3) Expand around a minimum of the Higgs potential to show that the Lagrangian of the $U(2)$ Higgs model describes a massless photon, three massive spin particles and a neutral spin zero particle. Derive formulae for the masses in terms of the constants in the Lagrangian.

$$\nabla_\mu \phi = \partial_\mu \phi + iL_\mu \phi + iqY_\mu \phi$$

$$L = \frac{1}{4e_1^2} \text{tr} L^{\mu\nu} L_{\mu\nu} + \frac{1}{4e_2^2} Y^{\mu\nu} Y_{\mu\nu} + \frac{1}{2} |\nabla\phi|^2 - V(|\phi|)$$