1. Problem Set 2 Due Oct 13 2010

1.1. Consider the metric $ds^2 = \frac{dx^2 + dy^2}{y^2}$ on the upper half plane.

1.1.1. Show that the transformation $x + iy \equiv z \mapsto \frac{az + b}{cz + d}$ is a symmetry of the metric if the parameters $a, b, c, d$ are real numbers with $ad - bc \neq 0$.

1.1.2. Solve the geodesic equation of this metric. (It will be useful to exploit conservation laws.)

1.2. Assume that the metric of space-time in a weak gravitational field $\frac{|\phi|}{c^2} << 1$ is $ds^2 \approx h_{1} + 2\phi(x) c^2 dt^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$. Use the variational principle to derive the equation of motion of particles, in the approximation that the velocity is small compared to $c$. Compare to the equations of Newtonian gravity.

1.3. A particle of mass $m$ and charge $q$ is moving in a gravitational field given by the metric $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ as well as an electromagnetic field with potential $A_\mu$. What is the action principle of this system? What are its equations of motion?