PHY114 S11 Term Exam 1

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12:30 pm to 1:45 pm

PLEASE write your workshop number and your workshop leader's name at the top of your book, so that you can collect your graded exams at the workshop.

Derive a formula for the answer before you put in the numbers. This will help you get partial credit if your final numerical answer is wrong.

Put a box around your final answer for each question, so that it does not go unnoticed by the grader.

Give answers to two significant digits.

The magnitude of the charge on an electron is $1.60 \times 10^{-19}C$

The mass of the electron is $9.11 \times 10^{-31}kg$.

The mass of the proton is $1.67 \times 10^{-27}kg$.

The permittivity of the vaccum is $\epsilon_0 = 8.85 \times 10^{-12}C^2N^{-1}m^{-2}$

Newton's Gravitational constant is $G = 6.67 \times 10^{-11}Nm^2kg^{-1}$.

$\pi = 3.1415927$.

1. (Grader: Kaminski) What is the magnitude of the electric force of attraction between an iron nucleus (atomic number 26) and its innermost electron, if the distance between them is $1.5 \times 10^{-12} m$? (10 points)

2. (Grader: Bose) There is an electric field at the surface of the Earth, pointing inward at every point, of magnitude $150 NC^{-1}$. Think of the Earth as a conducting sphere .

- What is the surface charge density of the Earth? (5 points)
- How many excess electrons per square meter on the Earth's surface should there be to produce this electric field?(5 points)

3. (Grader: Tepp) Consider two spheres of radii R_1 and R_2 ; their centers are at a fixed, large distance d apart; i.e., $d >> R_1, R_2$. Assume that the charges are distributed uniformly on the surface of the spheres.

- To begin with, the first sphere carries a charge Q and the other has no charge on it. What is the force between them? (2 points)
- A charge Q_2 is transferred from the first sphere to the second sphere. What is force between them now? (3 points)
- What should be the value of Q_2 in order that the magnitude of this force is as large as possible? Prove your answer. (5 points)

4. (Grader: Viza) A thin cylindrical shell of radius R_1 is surrounded by a second cylindrical shell of radius R_2, as in the figure . Both cylinders are of length L , which is much greater than their radii. The inner cylinder carries a total charge Q_1 and the outer one a charge Q_2.

- What is the electric field at a distance r from the central axis, when $R_1 < r < R_2$? (3 points)
- What is the electrical potential difference between a point on the inner cylinder and a point on the outer cylinder?(3 points)
- If an electron of charge q and mass m escaped the inner cylinder with negligible speed, what would be its speed when it reaches the outer cylinder?(4 points)

Figure 1:

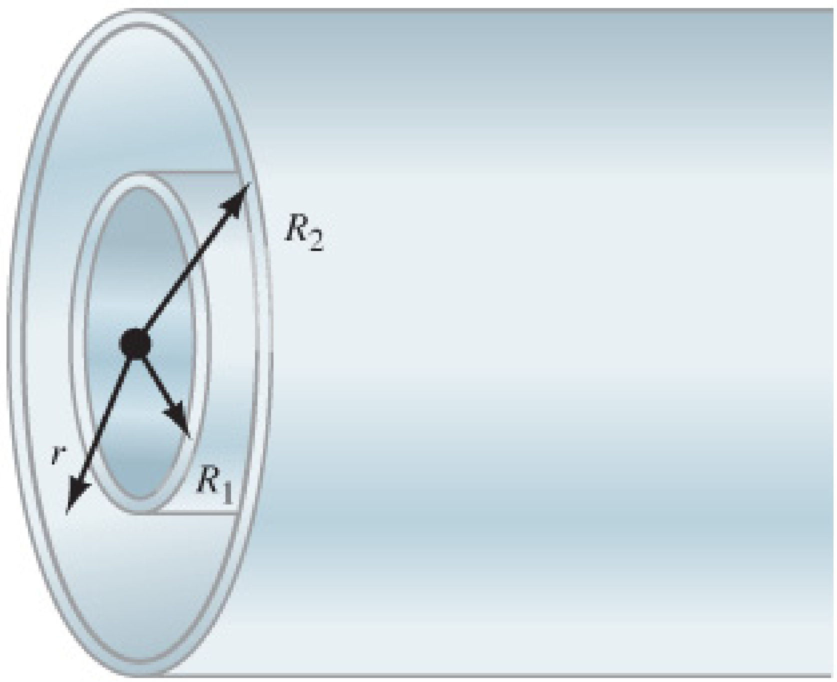

Figure 2:

Solutions

1. Straightforward use of Coulomb's Law:

$$F = k\frac{Q_1 Q_2}{r^2} = 9.0 \times 10^9 \frac{26 \times \left[1.6 \times 10^{-19}\right]^2}{\left[1.5 \times 10^{-12}\right]^2} = 2.7 \times 10^{-3} N.$$

2. The charge on a conducting sphere is on its surface. But the electric field outside it is the same as if all the charge is concentrated at the center. For a sphere of radius r and charge Q,

$$E = k\frac{Q}{r^2}$$

$$Q = \frac{1}{k} r^2 E$$

The surface charge density is

$$\sigma = \frac{Q}{4\pi r^2} = \frac{E}{4\pi k} = \epsilon_0 E$$

Note that the radius cancels out. The field points inward so this charge density is negative. The number of electrons is

$$N = \frac{|\sigma|}{e}$$

where e is the magnitude of the charge on the electron. Putting in the numbers

$$\sigma = -8.85 \times 10^{-12} \times 150 = -1.33 \times 10^{-9} Cm^{-2}.$$

$$N = \frac{1.33 \times 10^{-9}}{1.60 \times 10^{-19}} = 8.3 \times 10^{9} m^{-2}.$$

3. The spheres have the same electric fields as point charges at their centers. So the radii R_1, R_2 are irrelevant.

- When one of the spheres has no charge on it, there is no force between them.

- The total charge is Q; the charge on the second sphere is Q_2. Thus the charge of the first sphere is $Q - Q_2$. By Coulomb's law, the force is repulsive and its magnitude is

$$F = k\frac{(Q - Q_2)Q_2}{d^2}.$$

- Since $d$is fixed, the force is a function only of Q_2.To be be a maximum the derivative with respect to Q_2 must vanish:

$$Q - 2Q_2 = 0$$

Hence the force is a maximum when $Q_2 = \frac{1}{2}Q$; i.e., when the charge is equally distributed between the spheres.

4. Imagine a cylinder of radius r in between R_1 and R_2. The electric field is normal to its surface and has constant magnitude. So the electric flux is $2\pi r L E(r)$. By Gauss' law

$$2\pi r L E(r) = \frac{Q_1}{\epsilon_0},$$

so that

$$E(r) = \frac{Q_1}{2\pi\epsilon_0 r L}.$$

Note that this is independent of the charge on the outer cylinder.
The electric potential is related to the Electric field by

$$E(r) = -\frac{dV}{dr}$$

Since

$$\frac{d}{dr}[\log r] = \frac{1}{r}$$

we have

$$V(r) = -\frac{Q_1}{2\pi\epsilon_0 L}\log r.$$

Thus the electrical potential difference between the cylinders is

$$V(R_1) - V(R_2) = -\frac{Q_1}{2\pi\epsilon_0 L}[\log R_1 - \log R_2] = \frac{Q_1}{2\pi\epsilon_0 L}\log\frac{R_2}{R_1}.$$

The initial kinetic energy of the electron is zero. The initial potential energy is $qV(R_1)$, where q is the charge on the electron. If T is the final kinetic energy,

$$T + qV(R_2) = qV(R_1)$$

$$T = q[V(R_1) - V(R_2)] = \frac{qQ_1}{2\pi\epsilon_0 L}\log\frac{R_2}{R_1}$$

This is related to the speed v and the mass of the electron by

$$\frac{1}{2}mv^2 = T.$$

$$v = \sqrt{\frac{2T}{m}} = \sqrt{\frac{qQ_1}{\pi\epsilon_0 mL}\log\frac{R_2}{R_1}}$$

Clearly, Q_1 must be negative for this to happen.