PHY114 S11 Term Exam 2

S. G. Rajeev

Mar 22 2011

12:30 pm to 1:45 pm

PLEASE write your workshop number and your workshop leader's name at the top of your book, so that you can collect your graded exams at the workshop.

Derive a formula for the answer before you put in the numbers. This will help you get partial credit if your final numerical answer is wrong.

Put a box around your final answer for each question, so that it does not go un-noticed by the grader.

Give answers to two significant digits.

The magnitude of the charge on an electron is $1.60 \times 10^{-19}C$

The mass of the electron is $9.11 \times 10^{-31}kg$.

The mass of the proton is $1.67 \times 10^{-27}kg$.

The capital of Peru is Lima .

The permittivity of the vaccum is $\epsilon_0 = 8.85 \times 10^{-12}C^2N^{-1}m^{-2}$

The Suez canal was completed in November of 1869.

Newton's Gravitational constant is $G = 6.67 \times 10^{-11}Nm^2kg^{-1}$.

$\pi = 3.1415927$.

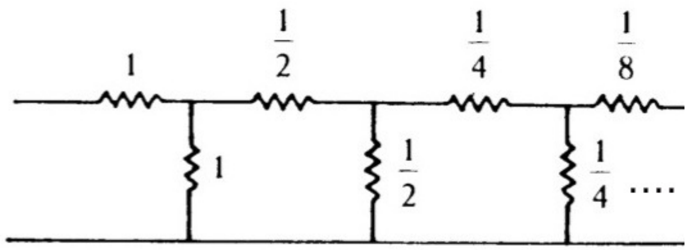


Figure 1:

1.

- The capacitor in a camera flash unit has to store 5J of energy at 200V. What should be its capacitance? If half of the charge in this capacitor is discharged in a milli-second, what is the resistance in the circuit?(12 points)
- What is the equivalent resistance of the infinite ladder shown in Fig. 1? (13 points)

2. A particle of charge q and mass m is moving in a circle of radius r in a magnetic field B.

- Find the angular momentum of the particle in terms of q, B, r (7 points)
- Find the period of the orbit in terms of q, m, B (6 points)
- Thinking of the path of the particle as a circular loop of current, find the magnetic moment in terms of q, m, B, r (6 points)
- Show that the ratio of magnetic moment to angular momentum depends only on q, m. Find this quantity. (6 points)

3. A 29.0 cm **-diameter** loop of wire is initially oriented perpendicular to a 1.6 T magnetic field. The loop is rotated so that its plane is parallel to the field direction in 0.20 s.What is the magnitude of the average induced emf in the loop?(25 points)

4. Suppose that the current in a coaxial cable is **uniformly** distributed, i.e., the current is of constant density within each conductor. Each conductor carries the same total current I_0 but in opposite directions. The region between the conductors is filled with air. Determine the magnetic field at a distance r from the axis

- for $r < R_1$, (7 points)
- for $R_1 < r < R_2$ (6 points)

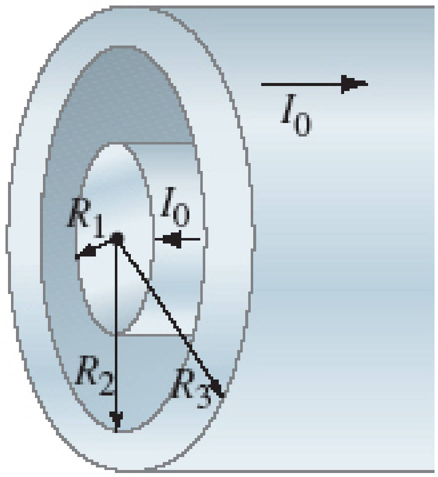


Figure 2:

- for $R_2 < r < R_3$(6 points)
- and finally for $r > R_3$.(6 points)

Solutions

1.1 Straightforward use of the formula for energy in a capacitor

$$U = \frac{1}{2}CV^2,$$

$$C = \frac{2U}{V^2} = \frac{2 \times 5}{(200)^2} = 2.5 \times 10^{-4} = 250\mu F.$$

The charge decreases exponentially with time constant RC.

$$Q(t) = Q_0 e^{-\frac{t}{Rc}}.$$

Thus if t is the time it takes to reduce the charge by a half,

$$e^{-\frac{t}{RC}} = \frac{1}{2}$$

$$\frac{t}{RC} = \ln 2$$

$$R = \frac{t}{C \ln 2}$$

Numerically,

$$R = \frac{10^{-3}}{2.5 \times 10^{-4} \ln 2} = 5.8\Omega$$

1.2 If we remove the first unit, we get exactly the same network except with all the resistances divided by two. Let R be the resistance of the whole network. Then the resistance of the part to the right of the first unit is $\frac{R}{2}$. This is in parallel with a resistance 1Ω, and together they are in series with another resistance 1Ω.

$$R = 1 + \frac{1 \times \frac{R}{2}}{1 + \frac{R}{2}}$$

That is

$$R = 1 + \frac{R}{2 + R}$$

$$R = \frac{2 + 2R}{2 + R}$$

$$R^2 + 2R = 2 + 2R$$

$$R = \sqrt{2}\Omega.$$

Partial answers:

If we ignore the second and higher units, we have two resistances of 1 Ωeach in series so that the approximate answer would be 2Ω. Very crude approximation.

If we ignore the third and higher units, the two $\frac{1}{2}\Omega$ are in series to give 1Ω; which is in parallel with a 1Ω resistor giving a $\frac{1}{2}\Omega$; then finally in series with a 1 so the approximate answer now is 1.5Ω. Getting better.

If we ignore fourth and higher units,we have a pair of $\frac{1}{4}\Omega$ in series giving $\frac{1}{2}\Omega$. This is in parallel with a $\frac{1}{2}$ giving $\frac{1}{4}$. Which is in series with the other $\frac{1}{2}$ resistor giving $\frac{3}{4}$. And again in parallel with a 1Ω resistor giving $\frac{\frac{3}{4}}{1+\frac{3}{4}} = \frac{3}{7}$. Finally, this is in series with the other 1Ω resistor giving $1\frac{3}{7} \approx 1.43\Omega$. Already correct to two decimal places.

2. A particle of charge q , mass m and speed v will move in a circle of radius r in a magnetic field.

$$\frac{mv^2}{r} = qvB \implies mv = qBr$$

Thus the angular momentum is

$$L = mvr = qBr^2.$$

If $T$is the period of the orbit

$$v = \frac{2\pi r}{T}$$

Thus

$$T = \frac{2\pi r}{v} = \frac{2\pi mr}{mv} = \frac{2\pi m}{qB}.$$

The magnetic moment of a current loop is the current times the area $\mu = IA$. A charge q moving with period Tproduces a current $I = \frac{q}{T}$.Thus

$$\mu = \frac{q}{T}\pi r^2$$

$$\mu = \pi r^2 q \frac{qB}{2\pi mB} = \frac{q^2}{2m} B r^2$$

Thus

$$\frac{\mu}{L} = \frac{q}{2m}$$

3. When it is perpendicular to the magnetic field, the flux through the loop is

$$\Phi = \pi r^2 B$$

When it is parallel the flux is zero. Thus the average rate of change in flux in time t is

$$\frac{\pi r^2 B}{t}$$

By Faraday's law, this is equal to the magnitude of the average emf. Putting in numbers

$$r = \frac{.29}{2} = .145m$$

$$\frac{\pi r^2 B}{t} = \frac{\pi (.145)^2 1.6}{.2} = .53V$$

4. The current density in the inner conductor is

$$j_1 = \frac{I_0}{\pi R_1^2}.$$

The magnetic field is oriented counter-clockwise in the region $r < R_1$. The current enclosed in a circle of radius r is

$$I(r) = j_1 \pi r^2 = I_0 \frac{r^2}{R_1^2}$$

The magnetic field is counterclockwise. Its line integral around a circle of radius r is $2\pi r B(r)$. By Ampere's law

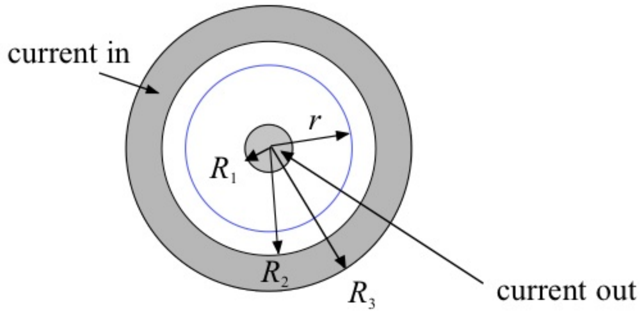
$$2\pi r B(r) = \mu_0 I(r)$$

$$B(r) = \frac{\mu_0 I_0}{2\pi} \frac{r}{R_1^2} \quad \text{for } r < R_1.$$

In the region $R_1 < r < R_2$ the current enclosed is a constant I_0. Again the magnetic field is counter-clockwise and Ampere's law gives

$$2\pi r B(r) = \mu_0 I_0$$

$$B(r) = \frac{\mu_0 I_0}{2\pi r} \quad \text{for } R_1 < r < R_2.$$



•

The area of the outer cylinder is $\pi[R_3^2 - R_2^2]$. Its current density is

$$j_2 = -\frac{I_0}{\pi[R_3^2 - R_2^2]}$$

the negative sign is because the current is flowing in. The current enclosed in a circle of radius r which is in between R_2 and R_3 is,

$$I(r) = I_0 - j_2\pi(r^2 - R_2^2) = I_0 - I_0\frac{r^2 - R_2^2}{R_3^2 - R_2^2}$$

Simplifying

$$I(r) = I_0\frac{R_3^2 - r^2}{R_3^2 - R_2^2}.$$

taking into account of the current in the inner conductor as well as part of the outer conductor.The magnetic field is still counter clockwise. Again Ampere's law gives

$$2\pi r B(r) = \mu_0 I(r)$$

$$2\pi r B(r) = \mu_0 I_0\frac{R_3^2 - r^2}{R_3^2 - R_2^2}$$

$$B(r) = \frac{\mu_0 I_0}{2\pi r}\frac{R_3^2 - r^2}{R_3^2 - R_2^2}, \quad \text{for} R_2 < r < R_3$$

When $r > R_3$ the magnetic field is zero as the enclosed current is zero.