PHY114 S11 Term Exam 3

S. G. Rajeev

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12:30 pm to 1:45 pm

PLEASE write your workshop number and your workshop leader's name at the top of your book, so that you can collect your graded exams at the workshop.

Derive a formula for the answer before you put in the numbers. This will help you get partial credit if your final numerical answer is wrong.

Put a box around your final answer for each question, so that it does not go un-noticed by the grader.

Give answers to two significant digits.

The magnitude of the charge on an electron is $1.60 \times 10^{-19} C$

The mass of the electron is $9.11 \times 10^{-31} kg$.

The mass of the proton is $1.67 \times 10^{-27} kg$.

The speed of light is 2.998 $\times 10^8$ ms^{-1}

The permittivity of the vaccum is $\epsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

The permeability of the vacuum is $\mu_0 = 1.258 \times 10^{-6}$ Hm^{-1}

Isaac Newton was born on Christmas Day of 1642.

$\pi = 3.1415927$.

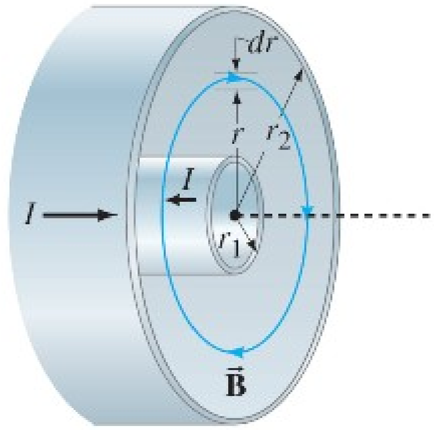

Figure 1:

1.

- A coil has 3.30 Ω resistance and 400 mH inductance. If the current is 3.89 A and is increasing at a rate of 3.62 As^{-1}, what is the potential drop across the coil at this moment?(8 points)
- An alternating current of peak value of 1.5A and frequency 46Hz is passed through a coil with inductance of $392mH$. What is the peak voltage? Ignore the resistance.(8 points)
- You are given two inductors $L_1 = 200mH, L_2 = 300mH$ and two capacitors $C_1 = 1.00\mu F, C_2 = 2.2\mu F$. How would you connect them up to get a circuit with the largest possible natural frequency of oscillation? What is this frequency? Ignore resistance. (9 points)

2. Determine the total energy stored per unit length in the magnetic field **between** the conductors of a coaxial cable carrying a current I in each direction (see Fig.1). (15 points.)What is the inductance per unit length of this coaxial cable?(10 points.) Ignore the magnetic field inside the conductors. Also, recall that the energy density of a magnetic field is $\frac{|\mathbf{B}|^2}{2\mu_0}$.

3.

- Suppose a $39kW$ radio station emits EM waves uniformly in all directions.How much energy per second crosses a 0.6 m^2 area 1 km from the transmitting antenna?(8 points)
- What is the range of wavelengths for FM radio (88MHz to 108MHz)? What is the range of wavelengths for AM radio (535 kHz to 1700 kHz)? (9 points)
- A light beam coming from an underwater spotlight exits the water at an angle of 56.6° to the normal. At what angle of incidence did it hit the

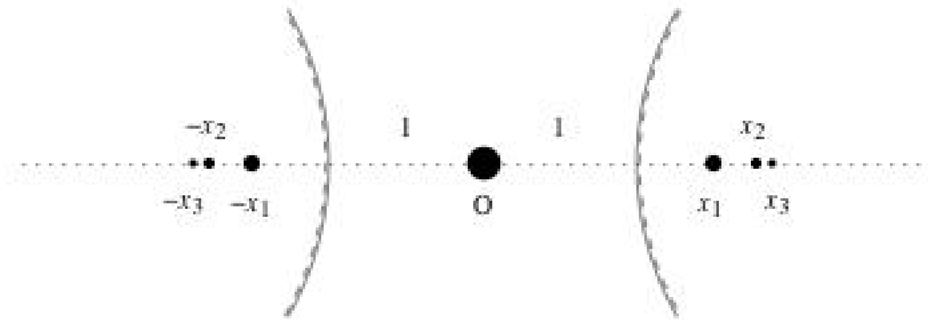

Figure 2:

air-water interface from below the surface? The refractive index of water is 1.33. (8 points)

4.

- A small candle is 35 cm from a convex lens of focal length 12 cm . Where is the image located? Is it real or virtual? Is it larger or smaller? Is it inverted or upright?(8 points)
- The magnification of the image in a convex mirror is m, the distance to the object is d. What is the radius of curvature of the mirror?(8 points)
- Suppose we place two convex mirrors of equal radius of curvature 2 meters , facing each other at a distance of 2 meters also. An object is placed at the midpoint in between the mirrors. What are the positions of the images formed by multiple reflections?What are the limit points to which the sequence of images tend?(**Hint** It is useful to choose a co-ordinate system whose origin is at the object.See Fig 2)(9 points)

Solutions

1.

- The potential difference is the sum of the those caused by the resistance and the inductance of the coil. Both cause a **drop** in the potential (the resistor always causes a drop and in our case the current is increasing, so the induced emf on the coil is negative as well) . Thus the potential drop across the coil is

$$RI + L\frac{dI}{dt}$$

$$= 3.3 \times 3.89 + .4 \times 3.62 = 14.3V$$

- Again, the induced emf is $-L\frac{dI}{dt}$. The current as a function of time is $I(t) = I_0 \sin \omega t$. Here I_0 is the peak current and $\omega = 2\pi f$ where f is the frequency. Thus

$$-L\frac{dI}{dt} = -LI_0\omega \cos \omega t$$

The peak voltage (also called emf) is $LI_0\omega = 2\pi LI_0 f = 2\pi \times .392 \times 1.5 \times 46 = 169.9V$.

- The natural frequency of an LC circuit is $\frac{1}{2\pi\sqrt{LC}}$.Thus to have a large frequency we need both a small inductance and a small capacitance. Two **inductances in parallel** have smaller inductance $L = \frac{L_1 L_2}{L_1+L_2}$ than each of them separately or in series (L_1+L_2) , we must connect them in parallel. Since **capacitances connected in series** have smaller capacitance $C = \frac{C_1 C_2}{C_1+C_2}$ than either of them in parallel $(C_1 + C_2)$ we must connect them in series. Thus

$$L = \frac{.2 \times .3}{0.2 + 0.3} = 120mH$$

$$C = \frac{2.2}{3.2} = .688\mu F$$

$$f = \frac{1}{2\pi\sqrt{.12 \times .688}} = 0.55Hz$$

2. The energy per unit volume of a magnetic field is $\frac{1}{2\mu_0}B^2$. The magnetic field has a magnitude that only depend on the radial distance. Thus the energy over some some length l of the cable will be given by integrating this density over the cross-section of the cable:

$$l\int_{r_1}^{r_2} \frac{1}{2\mu_0}B^2(r)2\pi r dr$$

Thus the energy per unit length is

$$\int \frac{1}{2\mu_0}B^2(r)2\pi r dr.$$

By Ampere's law

$$2\pi r B = \mu_0 I$$

Thus

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\frac{1}{2\mu_0} B^2 2\pi r = \frac{1}{2\mu_0} \frac{\mu_0^2 I^2}{2\pi r} = \frac{\mu_0 I^2}{4\pi r}$$

$$\int_{r_1}^{r_2} \frac{1}{2\mu_0} B^2(r) 2\pi r dr = \frac{\mu_0 I^2}{4\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I^2}{4\pi} \ln \frac{r_2}{r_1}.$$

The energy stored in an inductor is $\frac{1}{2}LI^2$. The energy per unit length is

$$\frac{1}{2}\frac{L}{l} I^2 = \frac{\mu_0 I^2}{4\pi} \ln \frac{r_2}{r_1}$$

We see that the inductance per unit length of the coaxial cable is

$$\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}.$$

Another Method

You can use Farday's law to calculate the inductance per unit length. The energy of the magnetic field per unit length is then $\frac{1}{2}\frac{L}{l}I^2$.

3.

- The power that crosses a sphere of radius r that surrounds the antenna must be equal to the power emitted by conservation of energy. Thus the power crossing a unit area is $\frac{P}{4\pi r^2}$.The power crossing an area A at a distance r is

$$\frac{P}{4\pi r^2} A = \frac{39 \times 10^3 \times 0..6}{4\pi [1000]^2} = 1.86 mW$$

- Just a matter of using $\lambda f = c$ where λ is the wavelength, f the frequency and c is the velocity of light.

$$\lambda = \frac{c}{f}$$

  so that at $88MhZ$, $\lambda = \frac{3}{.88} m = 3.41m$; at $108MHz$, $\lambda = \frac{3}{1.08} m = 2.78m$; at $.535MHz$ $\lambda = \frac{3}{.535\times10^{-2}} m = 560m$; at $1.7MHz$ $\lambda = \frac{3}{1.7\times10^{-2}} = 176m$.

- By Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Thus $\sin\theta_2 = \frac{n_1}{n_2} \sin\theta_1 = \frac{\sin 56.6^\circ}{1.33} = 0.6277$. Thus the angle of incidence is $\theta_1 = 38.9^\circ$.

4.

- The distance to the image i is given by

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f}, \implies i = \frac{df}{d-f} = \frac{35 \times 12}{35 - 12} = 18.3cm$$

  This is a real image located on the opposite side of the lens from the candle. The magnification $\frac{18.3}{35} < 1$ so the image is smaller. Also, it is inverted. (No need to draw diagrams if the answer is clearly explained)

- The magnification is related to the distance of the image i and the distance to the object d by $m = \frac{|i|}{d}$. For a convex mirror the image is always virtual and $i < 0$. Thus $i = -md$. The lens formula gives

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f} \implies f = \frac{id}{d+i} = \frac{-md^2}{d - md}$$

$$f = -\frac{m}{1-m}d$$

  Since $f < 0$, the magnification is always less than one. The radius of curvature is $R = 2|f| = \frac{2md}{1-m}$.

- It is useful to choose a co-ordinate system whose origin is at the object. Let x_1 be the position of the first image formed by the right mirror. By symmetry, the image formed by the left mirror will be at $-x_1$. Now the distance to the object from the right mirror is $d = 1$. The distance to the image is $(x_1 - 1)$ Since the image is virtual $i = -(x_1 - 1)$ .The focal length is $f = -1$. (Half the radius of curvature and negative because the mirrors are convex).Thus,

$$\frac{1}{1} - \frac{1}{x_1 - 1} = -1 \implies x_1 = \frac{3}{2}$$

  Similarly the left mirror forms an image at $-x_1 = -\frac{3}{2}$. For the second reflection, the object is at the point $-x_1$. So the distance to the object from the mirror is $d = 1 + x_1$.Again $f = -1$;the distance to the image from the mirror is $i = -(x_2 - 1)$

$$\frac{1}{x_1 + 1} - \frac{1}{x_2 - 1} = -1$$

  Similarly

$$\frac{1}{x_n + 1} - \frac{1}{x_{n+1} - 1} = -1$$

  As n becomes large $x_n \to x_{n+1}$so that the limiting position of the images satisfies

$$\frac{1}{x+1} - \frac{1}{x-1} = -1, \implies x = \pm\sqrt{3} \approx \pm 1.73$$

  Thus the images accumulate at two points 0.73 behind each mirror. **Special thanks to Emily Redman.**