PHY114 S11 Final Exam

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7:15 pm to 9:15 pm

PLEASE write your workshop number and your workshop leader's name at the top of your book, so that you can collect your graded exams at the workshop.

Derive a formula for the answer before you put in the numbers. This will help you get partial credit if your final numerical answer is wrong.

Put a box around your final answer for each question, so that it does not go un-noticed by the grader.

Give answers to two significant digits.

The magnitude of the charge on an electron is $1.60 \times 10^{-19} C$

The mass of the electron is $9.11 \times 10^{-31} kg$.

The mass of the proton is $1.67 \times 10^{-27} kg$.

The speed of light is $2.998 \times 10^{8}\ ms^{-1}$

The permittivity of the vaccum is $\epsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$

The permeability of the vacuum is $\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$

Plank's constant is $h = 4.13 \times 10^{-15} eVs$

$\pi = 3.1415927$

$\sin 2\theta = 2 \sin\theta \cos\theta$

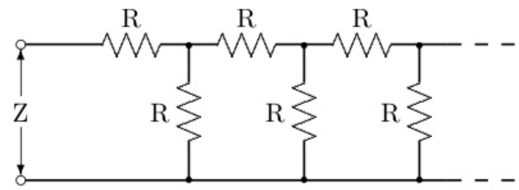

Figure 1:

1. What is the potential difference between the positions of an electron and a proton in a hydrogen atom, assuming that the distance between them is 53×10^{-12}m ? (8 points)

2. A pair of parallel plates carry equal and opposite charge densities and are at a distance d apart. Assume that the charge is distributed uniformly; i.e., the surface charge densities are constant. A particle of mass m and positive charge q is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate in a time interval t. What is its acceleration? What is the electric field between the plates? What is the magnitude of the electric charge per unit area on the plates? Express your answers in terms of q, m, d, t and constants of nature. (8 points)

3. A capacitor is charged to 7.5kV and stores 1300 J of energy. What is its capacitance?(8 points)

4. Find the effective resistance Z of the infinite ladder of resistors shown in the figure.(8 points)

5. An iron atom has a magnetic moment of about $1.8 \times 10^{-23} Am^2$. What torque would be exerted on this atom if is placed in a magnetic field of $0.8T$, acting at an angle $30°$ to the magnetic moment? (8 points)

6. A long pair of insulated wires serves to conduct 43.0 A of dc current to and from an instrument.If the wires are of negligible diameter but are 2.8mm apart, what is the magnetic field 10.0 cm from their midpoint, in their plane (see Figure 2)?(8 points)

7. An LCR circuit has inductance $4.69mH$, a capacitance $2.64 \times 10^{-9} F$ and a resistance $2.59k\Omega$. An alternating current of peak voltage $112V$ is applied to this circuit. What is the frequency at which the r.m.s. current is a maximum? What is the peak current at this frequency?(8 points)

8. An atomic nucleus of mass m and charge q moves in a constant magnetic field B; its path is a circle of radius r . What is its speed? Is it possible to make the particle move in a straightline by turning on in addition an electric field? What should be the direction and magnitude of this electric field?(8 points)

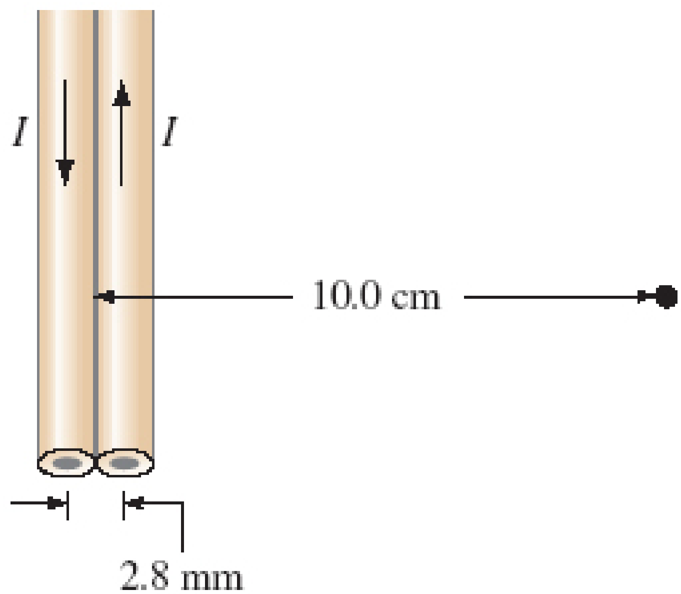

Figure 2:

9. Estimate the power per unit area of sunlight that hits Mars, knowing that the Earth receives about $1350 W m^{-2}$ and that the distance of Mars from the Sun is 1.5 times the distance of Earth from the Sun. (8 points)

10. A beam of light in air strikes a slab of glass with refractive index 1.60 and is partially reflected and partially refracted.Determine the angle of incidence if the angle of reflection is twice the angle of refraction. (8 points)

11. Sunlight is observed to focus at a point 20 cm behind a lens. What is the magnification when the object is at a distance of 10 cm? (8 points)

12. What is the speed of pions in a beam if their average lifetime is measured to be $5 \times 10^{-8}\ s$? At rest, the pion has an average lifetime $2.6 \times 10^{-8} s$. (8 points)

13. About 0.1 eV is required to break a "hydrogen bond" in a protein molecule.Calculate the maximum wavelength of a photon that can accomplish this. (4 points)

Solutions

1. We have,

$$V = k\frac{Q}{r}, \quad k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 Nm^2C^{-2}$$

$$Q = 1.6 \times 10^{-19}C$$

$$r = 53 \times 10^{-12}m$$

$$V = \frac{9 \times 10^9 \left[1.6 \times 10^{-19}\right]}{\left[53 \times 10^{-12}\right]} = 27.2 \text{ Volts}$$

2. The electric field in between two oppositely charged parallel plates, each of uniform charge density of magnitude σ is a constant

$$E = \frac{\sigma}{\epsilon_0}.$$

It points from the positive plate to the negative plate.This follows from Gauss' law. Thus, the force qE and acceleration a of a charged particle in this region are constant. If the particle starts at rest, the distance travelled in time t is

$$d = \frac{1}{2}at^2$$

Thus

$$a = \frac{2d}{t^2}.$$

So the force is

$$F = ma = \frac{2md}{t^2}.$$

The electric field is

$$E = \frac{F}{q} = \frac{2md}{qt^2}.$$

Thus the charge density is

$$\sigma = \epsilon_0 E = \frac{2\epsilon_0 md}{qt^2}.$$

3. The energy of the capacitor is $E = \frac{1}{2}CV^2$. Thus $C = \frac{2E}{V^2} = \frac{2\times1.3\times10^3}{[7.5\times10^3]^2} = 4.6 \times 10^{-5}F$

4.If we remove the first unit from the ladder, we get back itself. So the resistance of the part after the first unit is Z as well. This is connected in parallel to $R$and then again in series to R to get Z:

$$Z = R + \frac{RZ}{R+Z}.$$

That is

$$Z(R+Z) = R(Z+R) + RZ$$

$$Z^2 - RZ - R^2 = 0$$

$$Z = \frac{R \pm \sqrt{R^2 + 4R^2}}{2} = \frac{1 \pm \sqrt{5}}{2} R$$

The negative root does not make sense. So

$$Z = \frac{1 + \sqrt{5}}{2} R \approx 1.62 \ R$$

We can also solve this by a sequence of approximations. If we ignore all but the first unit, we just have two resistances in paralle, so

$$Z_1 = 2R.$$

If we include the second unit, we have first a pair of resistors in series to give $2R$; this is in parallel with R giving $\frac{2R \times R}{2R+R} = \frac{2}{3}R$; whicn in turn is in series with Rgiving

$$Z_2 = 1\frac{2}{3}R \approx 1.67 \ R$$

If we include also the third unit, again we start with two R's in series giving $2R$. This is again in parallel with R to give $\frac{2}{3}R$. Which is in series with R to give $1\frac{2}{3}R = \frac{5}{3}R$. This is in Parallel with R to give $\frac{\frac{5}{3}R \times R}{\frac{5}{3}R+R} = \frac{5}{8}R$. Finally this is in series with R to give the third approximation

$$Z_3 = 1\frac{5}{8}R \approx 1.625 \ R$$

And so on.

5. The torque is the cross product of the magnetic moment and the magnetic field. So it is in a direction perpendicular to both the magnetic moment and the field. Its magnitude is

$$\mu B \sin\theta = 1.8 \times 10^{-23} \times 0.8 \sin 30^\circ = .72 \times 10^{-23} Nm$$

6. For a single long wire carrying a current I, the magnetic field at a distance r is given by Amperes law to be

$$2\pi r B = \mu_0 I \implies B = \frac{\mu_0 I}{2\pi r}$$

In our case we have two wires carrying opposite currents, at a small separation δ. The magnetic fields they produce are in opposite directions so the net field at a distance r from one of them is

$$\frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi(r+\delta)}$$

We could now plus in the numbers and calculate the magnetic field. But it is easier to realize that δ is much smaller than r so that

$$\frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi(r+\delta)} \approx \frac{\mu_0 I}{2\pi r^2}\delta$$

Putting in the numbers,

$$\frac{4\pi \times 10^{-7}(43)}{2\pi(0.1)^2} 2.8 \times 10^{-3} \approx 2.4 \times 10^{-6} T$$

We can also directly calculate the value of the magnetic field from each wire and subtract, without using the derivative of $\frac{1}{r}$.

7. An LCR circuit will have the maximum r.m.s. current when it is at resonance; i.e., when the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{4.69 \times 10^{-3} \times 2.64 \times 10^{-9}}} = \frac{10^6}{2\pi\sqrt{4.69 \times 2.64}} = 45.2 kHz$$

At this frequency, the inductance and the capacitance exactly cancel each other so that the peak current is simply the peak voltage divided by the resitance

$$\frac{112}{2.59 \times 10^3} = 43.2 mA$$

8. The speed of a charged particle in a circular orbit in a magnetic field is given by equating the centripetal acceleration to the force:

$$m\frac{v^2}{r} = qvB$$

Thus

$$v = \frac{qBr}{m}.$$

If an electric field is applied in a direction perpendicular to the magnetic field, it is possible to cancel the electric and magnetic forces

$$q\mathbf{E} + q\mathbf{v} \times \mathbf{B} = 0$$

$$|E| = vB = \frac{qB^2 r}{m}.$$

Then the particle will move in a straightline perpendicular to both the electric and magnetic fields.

9. By conservation of energy, the power per unit area due to a point source at a distance r must be proportional to $\frac{1}{r^2}$: the total power crossing the surface area of a sphere of radius r must be independent of r. Thus

the power of light at a distance 1.5 times greater will be $\frac{1}{(1.5)^2} = .44$ times smaller. Thus the power per unit area at Mars is $0.44 \times 1350 = 600 W m^{-2}$.

10. The angle of incidence is equal to the angle of reflection. Let it be θ_1. By Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where θ_2 is the angle of refraction. When $\theta_1 = 2\theta_2$ and $n_1 = 1$ we have

$$\sin 2\theta_2 = n_2 \sin\theta_2$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$

$$2\cos\theta_2 = n_2$$

$$\cos\theta_2 = \frac{n_2}{2} = .8$$

Thus

$$\theta_2 = 36.8^\circ$$

And the angle of reflection is

$$\theta_1 = 2\theta_2 = 73.7^\circ$$

11. The lens is converging and of focal length $f = 20$cm.If the distance to the object is $d = 10$cm the distance to the image is given by

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f}$$

$$\frac{1}{i} = \frac{d-f}{df}$$

$$i = \frac{df}{d-f}$$

The magnification is the ratio $|m| = \frac{|i|}{d} = \frac{|f|}{|d-f|}$. The image is virtual and the magnification has magnitude

$$|m| = \frac{20}{20-10} = 2.$$

12. The lifetime of a particle moving at velocity v will be

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where T_0 is the lifetime at rest. So

$$1 - \frac{v^2}{c^2} = \frac{T_0^2}{T^2}$$

$$v = c\sqrt{1 - \frac{T_0^2}{T^2}}$$

$$= c\sqrt{1 - \left(\frac{2.6}{5}\right)^2}$$

$$= 2.6 \times 10^8 ms^{-1}$$

13. We have $hf = E$ and $\lambda f = c$ so that

$$\lambda = \frac{c}{f} = \frac{hc}{E}$$

$$\lambda = \frac{4.13 \times 10^{-15} \times 3 \times 10^8}{.1} \approx 12\mu m$$