PHY114 S11 Lecture 2: The Electric Field

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1 Vectors

Many physical quantities are represented as numbers: mass, temperature, charge. They can be positive or negative. Such quantities are calles *scalars*.

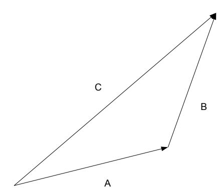
But there are other quantities, such as velocity that need both a number (its magnitude) and a direction to describe them fully. It is not enough to know how fast an airplane is moving, we have to also which direction it is moving to completely descrive its velocity. Such quantities are called *vectors*. Other examples of vectors are force, momentum and position.

It is useful to visualize a vector as a an arrow, pointing along some direction, with a length that is equal to its magnitude.

1.1 Adding Vectors

Adding scalars is easy: just add them as numbers. If we add a charge of $2\mu C$, to one of $-3.1\mu C$ we get a charge of $-1.1\mu C$.

Vectors can also be added but it is more tricky: the sum of two vectors points in some direction that is in between their two directions. To add two vectors we place them end to end. That is, we draw a line in the direction of the first vector $\mathbf{A}$ whose length is equal to its magnitude. From its endpoint, draw another line in the direction of the second vector again, $\mathbf{B}$ with a length equal to its magnitude. The sum of the two vectors $\mathbf{C} = \mathbf{A} + \mathbf{B}$ is the line from the starting point of $\mathbf{A}$ to the endpoint of $\mathbf{C}$.



1.2 Mutlipying Vector by a Scalar

Given a vector $\mathbf{A}$, its product with any positive number x is another vector pointing in the same direction, but who magnitude is x times the magnitude of $\mathbf{A}$. If x is negative the story is the same, except that $x\mathbf{A}$ points in the opposite direction to $\mathbf{A}$.

1.3 Unit Vectors

The magnitude of a vector (its length) is denoted by $|\mathbf{A}|$. Thus $\frac{\mathbf{A}}{|\mathbf{A}|}$ is a vector of unit length pointing in the same direction as $\mathbf{A}$. Sometimes it is denoted as $\hat{\mathbf{A}}$. It contains only the information about the direction of $\mathbf{A}$, not its length.

1.4 The Position Vector

The position of any point is itself a vector: it is the vector connecting the origin of your co-ordinate system to that point. You can think of the position vector $\mathbf{r}$ as a way of finding where the point is: if you move a certain distance $|\mathbf{r}|$ from the origin along a certain direction $\hat{\mathbf{r}}$, you will get to the point.

The relative position of two points is the vector that starts at one and ends at the other. In terms of the position vectors $\mathbf{r}_1$and $\mathbf{r}_2$ of the two points, the relative position of 2 with respect to one is

$$\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1.$$

2 Vector Form of Coulomb's Law

We saw that the force on a charge Qdue to another Q_1 is equal to $k\frac{QQ_1}{|\mathbf{r}-\mathbf{r}_1|^2}$ where $|\mathbf{r}-\mathbf{r}_1|$ is the distance between them. What is the direction of this force? It is pointed along the line connecting the two charges. The vector $\mathbf{r}-\mathbf{r}_1$ is the relative position of 2 with respect to 1. Its magnitude is the distance r_{12} between them.Thus we can restate Coulomb's law more precisely as

$$\mathbf{F} = k\frac{QQ_1}{|\mathbf{r}-\mathbf{r}_1|^2}\widehat{\mathbf{r}-\mathbf{r}_1}.$$

If $Q_1$and Q_2 have the same sign, the force is repulsive and the vector $\mathbf{F}_{12}$ points away from 2 towards 1. If they have opposite charges, the force points towards 2. All this information is contained in the vector form of the law. Since

$$\widehat{\mathbf{r}-\mathbf{r}_1} = \frac{\mathbf{r}-\mathbf{r}_1}{|\mathbf{r}-\mathbf{r}_1|}$$

we can also write this as

$$\mathbf{F} = k\frac{QQ_1}{|\mathbf{r}-\mathbf{r}_1|^3}(\mathbf{r}-\mathbf{r}_1)$$

which is often more convenient.

3 Many Charges

Suppose a charge q is situated at some point $\mathbf{r}$ near two other charges Q_1 and Q_2 which are at $\mathbf{r}_1$and $\mathbf{r_2}$respectively. Each will exert a force on q. What is the total force? It is given by the sum of the two.

We must add them as vectors:

$$k\frac{qQ_1}{|\mathbf{r}-\mathbf{r}_1|^3}(\mathbf{r}-\mathbf{r}_1) + k\frac{qQ_2}{|\mathbf{r}-\mathbf{r}_2|^3}(\mathbf{r}-\mathbf{r}_2)$$

The idea is the same if we have many charges: we just add one term for each charge:

$$k\frac{qQ_1}{|\mathbf{r}-\mathbf{r}_1|^3}(\mathbf{r}-\mathbf{r}_1)+k\frac{qQ_2}{|\mathbf{r}-\mathbf{r}_2|^3}(\mathbf{r}-\mathbf{r}_2)+k\frac{qQ_3}{|\mathbf{r}-\mathbf{r}_3|^3}(\mathbf{r}-\mathbf{r}_3)+k\frac{qQ_4}{|\mathbf{r}-\mathbf{r}_4|^3}(\mathbf{r}-\mathbf{r}_4)+\cdots$$

Notice that the force on a charge is proportional to its own electric charge time a quantity that is determined by all the remaining charges and their positions :

It is useful to give the quantity inside the brackets a name: it is called the electric field.

$$q\left[k\frac{Q_1}{|\mathbf{r}-\mathbf{r}_1|^3}(\mathbf{r}-\mathbf{r}_1) + k\frac{Q_2}{|\mathbf{r}-\mathbf{r}_2|^3}(\mathbf{r}-\mathbf{r}_2) + k\frac{Q_3}{|\mathbf{r}-\mathbf{r}_3|^3}(\mathbf{r}-\mathbf{r}_3) + k\frac{Q_4}{|\mathbf{r}-\mathbf{r}_4|^3}(\mathbf{r}-\mathbf{r}_4) + \cdots\right]$$

4 The Electric Field

The force on a charge at any point is its electric charge times the *electric field* at that point. Thus, the electric field at some point $\mathbf{r}$ due to a single charge Q sitting at the origin is $\frac{kQ}{r^2}\hat{\mathbf{r}}$ or, equivalently,

$$k\frac{Q}{r^3}\mathbf{r}.$$

If Q_1 is sitting at some point $\mathbf{r}_1$, the electric field at the point $\mathbf{r}$ will be

$$k\frac{Q_1}{|\mathbf{r}-\mathbf{r}_1|^3}(\mathbf{r}-\mathbf{r}_1)$$

This is another way of stating Coulomb's law. The electric field at the point $\mathbf{r}$ is the sum of the electric fields due to all the charges around it:

$$\mathbf{E}(\mathbf{r}) = k\frac{Q_1}{|\mathbf{r}-\mathbf{r}_1|^3}(\mathbf{r}-\mathbf{r}_1)+k\frac{Q_2}{|\mathbf{r}-\mathbf{r}_2|^3}(\mathbf{r}-\mathbf{r}_2)+k\frac{Q_3}{|\mathbf{r}-\mathbf{r}_3|^3}(\mathbf{r}-\mathbf{r}_3)+k\frac{Q_4}{|\mathbf{r}-\mathbf{r}_4|^3}(\mathbf{r}-\mathbf{r}_4)+\cdots$$

Thus we can imagine at every point in space a little arrow pointing in the direction of $\mathbf{E}(\mathbf{r})$: if you were to put a charge q at that point, the force acting on it would be $q\mathbf{E}(\mathbf{r})$.

Gauss found a way to restate Coulomb's law in a form that is more useful to calculate the electric field at any point.

This *Gauss' Law* is the topic of the next Lecture.