7. Dielectrics and Capacitors

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Some materials contain electrons that can move freely inside them. They are conductors; metals are examples. Others have all electrons tightly bound to atoms or molecules; they are insulators. A particular class of insulators are made of molecules that stretch a little in an electric field: the positive and negative charges moving in opposite directions. They are called dieletrics. They are useful in devices that can store electrical charge differences, called capacitors.

1 Capacitors

The simplest capacitor is a pair of metal plates with a vaccum in between. We can move electrons from one plate to the other using a conducting wire and a battery: the battery provides the potential energy needed to make electrons separate from each other. The plate that is depleted of electrons will have some positive charge Q and the other plate has charge $-Q$. If we now disconnect the wires the capacitor can remain in this state for a long time, serving as a store of electrical energy.

The electric field outside the capacitors is zero. Inside it is equal to (by Gauss' Law)

$$E = \frac{Q}{\epsilon_0 A}$$

where A is the area. Strictly speaking this formula is only valid if you stay away from the edges of the plates. If the distance between the plates is d the magnitude of the potential difference between the two plates is

$$V = Ed = \frac{Q}{\epsilon_0}\frac{d}{A}.$$

The capacitance is the ratio:

$$C = \frac{Q}{V}.$$

For a rectangular capacitor

$$C = \epsilon_0 \frac{A}{d}.$$

It only depends on the shape of the capacitor. The larger the capacitance, the more charge you can separate for a given potential difference.

1.1 Other Shapes

Using Gauss's law one can calculate the capacitance of other shapes. If you have two conducting cylinders with the same axis, with the inner cylinder of radius R_1 outer cylinder or radius R_2 charged to the opposite value, the capacitance is

$$C = \frac{2\pi\epsilon_0 l}{\log\frac{R_2}{R_1}}.$$

For two spheres of the same center and radii $R_1 < R_2$

$$C = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}.$$

If $R_2 \to \infty$ we just have a single sphere with capacity

$$C = 4\pi\epsilon_0 R_1.$$

Capacity is simply a measure of easily a conductor can hold a charge difference from its surroundings from which it is insulated. If it takes a lot of energy to do that the capacitance will be small.

2 Capacitors in Parallel and Series

To get larger capacitance, we can string together several of them in parallel. This means that each pair of plates is at the same potential difference V. If each has a charge Q_1, Q_2 the total charge is

$$Q = Q_1 + Q_2 = VC_1 + VC_2 so$$

that

$$C = \frac{Q}{V} = C_1 + C_2.$$

If capacitors are combinded in series, they are all at the same charge difference, but each has a different potential across them. Then $Q = C_1V_1 = C_2V_2$.Then the total potential difference is

$$V = V_1 + V_2 = Q\left[\frac{1}{C_1} + \frac{1}{C_2}\right]$$

and the net capacitance is

$$C = \frac{Q}{V} = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1} = \frac{C_1C_2}{C_1 + C_2}.$$

3 Energy in a Capacitor

If we add a charge dQ to the capacitor plate, its energy changes by

$$dW = VdQ = \frac{Q}{C}dQ$$

Integrating,

$$W = \frac{Q^2}{2C}.$$

So the larger the capacity, the less energy it takes to store charge in it. If you were to connect the ends of a capictor with a conducting wire, the electrons will move through the wire and cancel the charge out. The energy stored will be first converted to kinetic energy of the electrons and then to heat in the wire.

4 Dielectrics

It is impractical to have a vacuum separating charged plates in the real world. But we can put any insulating material to keep the electrons apart. If we chose the right material we can even increase the capacitance without increasing the size of the capacitor. Such materials are called dielectrics.

These are made of molecules that change their shape and orientation slightly when an electric field is applied. The positive charges are pushed in one direction and the negative charges (electrons) in the other. If the field is not too strong, the material remains insulating: the electrons are stuck to the molecules, only shifted slightly in position.

But this has a useful effect: it partially cancels out the electrical field in between the conductors. Or, it converts electrical energy into the energy of deforming the molecules.

The dielectric constant measures the factor by which the capacitance is increased by a dieletric filling the region between plates. It is a propetry of the filling material. The capacitance of a pair of parallel plates for example, is now

$$C = K\epsilon_0 \frac{A}{d}$$

The effect is simply to replace the constant ϵ_0 by $\epsilon = K\epsilon_0$. In the old days, people used to not be able to do experiments in the vacuum. So what they measured was this ϵ which had a different value for each substance, and is called *electrical permittivity.* The larger the permittivity the better the material as a filler for capacitors. Air has a dielectric constant close to one. Water has $K = 80$. Many plastics are used as fillers in capacitors.

If the applied eletric field is strong enough any dielecric will break down: the electrons will be ripped away from their molecules. The electric fields needed are large: for air it is about $3 \times 10^6 Vm^{-1}$. Or about 3000 Volts across one millimeter. When it happens we see a sparks, as the electrons emit light as they steak across. Lightning from thunderclouds is an example of this.