8. Electric Currents

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An electric current is produced by the movement of electric charges. In most cases these are electrons. A conductor is a material through which an electric charge can move easily. Metals are condutors, copper being the most commonly used to make electrical wires.

Consider electrons moving through a copper wire. Even in a good conductor like copper they will encounter many obstacles to their movement: other electrons, and atoms get in the way. So unlike in empty space, an electron does not keep moving once it starts: it is brought to a stop by friction, much like a person caught in a crowd. To keep an electrons moving we must exert a force on it, usually with an electric field.

If we connect the two ends of the wire to the poles of a battery, we can create a potential difference and hence an electric field inside the wire. The electrons will then be able to move to the positive end of the batter from from the negative end, completing a circuit. The larger the potential difference, the larger the current.

1 Ampere

To make matters more precise we will need a unit of electric current. One way would be to just count how many electrons cross some point of the wire per unit time. It would be like couting how many cars go by a bridge with one way traffic. That is likely to be a huge number so we instead measure how many Coulombs of charge cross a point per sec. This is the unit of elecric current called an *Ampere;* often abbreviated to amp or just A.

While one Coulomb is a huge amount of free charge, it is not unusual for one Coulomb (about 6×10^{19} electrons) to go through a wire in a second. A car battery provides about 200 A of current for few seconds to start the car.

Electric current is usually denoted by the letter I, just as electric potential is denoted by V.

2 Ohm's Law

The larger the potential difference the larger the current passing through a conductor. To a good approximation they are proportional:

$$V = RI$$

The constant of proportionality R is a property of the conductor: it is called resistance. It depends on the length and area as well as on the material it is made of.

An insulator has very large resistance, while good conductors like metals have low resistance.

The unit of resistance is *Ohm,* often abbreviated to Ω (the Greek letter Omega, but pronounced Ohm in this context.)

$$\text{Ohm} = \frac{\text{Volt}}{\text{Amp}}.$$

The human body has a resistance of about a 1000 Ω; men usually have lower resistance than women, as thicker arms and legs allow passage of electricity easier. A current of 10 A is usually lethal. So even a potential difference of a 100 V can give you an unpleasant electrical shock.

Ohm's Law is Not A Fundamental Law of Nature Unlike Coulomb's law or Gauss' law, Ohm's law is not a fundamental law of nature. It is merely an approximation. If you pass a large enough current you will heat the wire and its resistance will go up, so V and I are not strictly proportional. Gauss's law is however, a fundamental law which always holds: we have not found even the slightest variation from it in a hundred and fifty years of careful observation.

3 Power

Suppose there is a current I passing through a wire; the two ends of the wire are at a potential difference of V. Now recall that when a charge Q moves over a potential difference V it gains an energy of QV. Current is just the amount of charge moving across the condutor per unit time. Thus the amount of energy gained by the charges per unit time is IV : this is the power.

Now, this power is dissipated or wasted: it is turned into heating the conductor. There is no simple way of converting it back into electricity. In fact electrical resistance is caused by this conversion of electrical energy into heat.

The power dissipated in a conductor is, using Ohm's law

$$P = IV = RI^2 = \frac{V^2}{R}.$$

At a fixed current the higher the resistance the greater the dissipated power; at a fixed voltage, power is greater for low resistance.

4 Alternating Current

Most household devices (unlike portable ones) run on a alternating current. In these devices, the current is not a constant: it even reverses direction many times a second. The current is driven in one direction then the other, so that the current is a periodic function of time. The period is $\frac{2\pi}{\omega}$; the frequency is $\frac{\omega}{2\pi}$. The voltage

$$V(t) = V_0 \sin \omega t$$

driving the current is also a function of time, and the resistance is also a function of time. The resistance of appliances does not change with time.

The frequency of the electrical grid in the US is 60 Hz; that is the current reverses itseld 60 times a second. The peak voltage V_0 at an outlet is $170V$; people usually refer instead to the r.m.s. (square root of the average of the square of voltage) which is $120V$. In other countries (Europe and Asia) it is usually 50 Hz and an rms voltage of 240V.

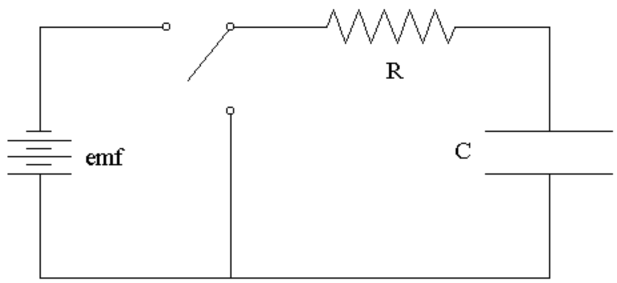
Alternating current is more convenient to transport over an electrical grid: it is possible to change the voltage of AC using a device called a transformer. This is harder do with direct current (current that is constant in time). The large electrical towers that bring the electricity into a city from a power plant are at very high voltage (120kV or more). This would be too dangerous to use in homes, so transformers are used to step it down to a lower voltage in several stages.

Why would you want the voltage to be so high in the grid? It is a way to minimize the losses due to resistance in the electrical transmission wires while still supplying a constant amount of power to the consumers.

We will see how transformers work later in the course.

5 RC Circuits

Most electrical devices have a source of energy (battery or power supply connected to an eletrical grid) that drives an electrical current through a complete loop or circuit. The simplest such device is a circuit consisting of a capacitor and a resistor . We can also add a switch that connects the device to a battery so that the capacitor can be charged; or allow it the capacitor to discharge bypassing the battery. If there were no resistance, the capacitor would discharge instantaneously. But in reality, it takes a finite time that is proportional to the proportional to the product of resistance and capacitance.



6 Discharging the Capacitor

Suppose that the capacitor has been charged so that each side carries an electrical charge Q_0. Remember that the total charge on a capacitor is zero; Q is the amount of charge that has been moved from one plate to the other. If we now connect the positive side of the capacitor to the negative side through a resistance R, a current will flow until there is no more charge left. We want to determine the charge $Q(t)$ on the capacitor at any instant t.

The potential difference between the two plates of a capacitor is

$$V(t) = \frac{Q(t)}{C}.$$

This will drive a current

$$I(t) = \frac{V(t)}{R} = \frac{Q(t)}{RC}.$$

But the current is simply the rate of transfer of the charge.

$$I(t) = -\frac{dQ}{dt}.$$

As the capacitor discharges, the current is in the direction that will decrease $Q(t)$.

Thus we get

$$\frac{dQ}{dt} = -\frac{Q}{RC}.$$

This is an example of a differential equation: the derivative of a quantity is related to itself.

The function whose derivative is equal to itself is the exponential.

$$\frac{d\ e^x}{dx} = e^x.$$

Notice also that

$$\frac{d\ e^{ax}}{dx} = ae^{ax}$$

for any constant a. Thus a solution to our equation is

$$Q(t) = e^{-\frac{t}{RC}}.$$

But it is not the only solution. If you multiply it by any constant you will get another solution. This is true of differential equations usually that they have many solutions and to pick the right one we need additional information. In our case, this is the knowledge of the charge at $t = 0$:

$$Q(0) = Q_0.$$

Thus

$$Q(t) = Q_0 e^{-\frac{t}{RC}}.$$

This tells you how fast the charge is discharged.

6.1 The Time Constant

The quantity

$$\tau = RC$$

that appears above has dimensions of time.

For a resistance of $R = 20k\Omega$, and a capacitance of $0.3\mu F$ (remember that a Farad is a very large unit of capacitance, so a micro-Farad is more the sort of capacitance you will encounter) the time constant is

$$\tau = 6ms$$

a few a milliseconds. This is about typical.

The smaller this time constant of the circuit the faster it will discharge. For example, the time time it takes to lose half the charge is given by

$$e^{-\frac{T}{\tau}} = \frac{1}{2}$$

or

$$T = \tau \log 2 \approx 0.69\tau.$$

This is called the half-life of the circuit. for the above example this is about $4ms$. In another $4ms$ the charge is reduced by half again; that is to a quarter of its original value. In $12ms$ it is reduced by another factor of two so that the charge an eighth of the original and so on. In principle, the charge never quite becomes zero: but in practice after a few half-lifes there is almost nothing left.

Such exponential decays are quite common in physics. If you have a radioactive material, the number of atoms that remain after time $t$is

$$N(t) = N_0 e^{-\frac{t}{\tau}}.$$

The time constant of the material again gives the half-life of the material as above.

7 Charging the Capacitor.

Suppose a capacitor has initial charge zero. If it is connected to a battery that supplies a constant potential difference $V_{0.}$,a charge will build up on the capacitor. The potential difference across the battery is equal in magnitude to the sum of the potential drop across the resistor and the capacitor

.

$$V_0 = RI(t) + \frac{Q(t)}{C}.$$

The current is in the direction that increases the charge:

$$I(t) = \frac{dQ}{dt}.$$

Thus we get another differential equation

$$R\frac{dQ}{dt} + \frac{Q}{C} = V_0.$$

This one is a bit harder because the r.h.s. is not zero. We already know the solution when $V_0 = 0$: it is proportional to $e^{-\frac{t}{RC}}$.

So let us turn the equation into what we know by rewriting it as

$$R\frac{dQ}{dt} + \frac{Q - CV_0}{C} = 0$$

If we define $Q_1(t) = Q(t) - CV_0$we get a familiar equation

$$R\frac{dQ_1}{dt} + \frac{Q_1}{C} = 0.$$

Thus

$$Q_1(t) = Ae^{-\frac{t}{\tau}}$$

$$Q(t) = CV_0 + Q_1(t) = CV_0 + Ae^{-\frac{t}{\tau}}.$$

We still have to determine A. In this case the initial charge is zero, $Q(t) = 0$.

$$CV_0 + A = 0$$

so that

$$Q(t) = CV_0 \left[1 - e^{-\frac{t}{\tau}} \right].$$

This shows a charge that starts zero and grows to a maximum value of CV_0.

8 Simplifying a Circuit

As long as we only have capacitors and resistors, any circuit can be reduced to the above simple one by repeated use of the rules for combining them in parallel and series. Or we can use Kirchhoff's rules. There are many examples in the end of the textbook.

By adding a switch that turns on when the voltage exceeds a critical value, we can produce a sawtooth waveform: the current grows to a maximum, then drops and starts to grow again. Most useful devices contain additional components such as inductors. We will study them later.

References

[1] The diagram is from http://biology.unm.edu/toolson