11. Sources of Magnetic Fields

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1 Magnetic Field Due to a Straight Wire

We saw that electric currents produce magnetic fields. The simplest situation is an infinitely long, thin, straight wire carrying a constant current I. The magnetic field peroduced has strength

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}.$$

Here r is the distance from the wire. That is, the distance from the point where B is being measured to the nearest point on the wire. μ_0 is a constant called the magnetic permeability of vacuum. (The formula above is strictly speaking true only in a vacuum, but it works well enough in air.) The value of this constant is

$$\mu_0 = 4\pi \times 10^{-7} TmA^{-1}.$$

Recall that the unit of magnetic field is Tesla (T) and that of current is Ampere (A).

1. This constant μ_0 is similar to the electric permittivity of the vaccum, ϵ_0. But it is *not* the same thing. Also, just to make our life complicated, the constant μ_0 appears in the numerator here while ϵ_0 is in the denominator of the similar formula for electric field. These definitions are left over from old times, and we are stuck with them.

2. Notice that this is similar to the formula for the elecric field produced by a charged wire. *But the direction of the magnetic field is different.*

   If your right thumb points along the current,

   the magnetic field curls around the wire

   in the direction of your fingers.

1.1 Example: Magnetic Field Produced by a Power Line

A power line carries a current of 97 A west along the tops of 9.0 m-high poles. What is the direction and the magnitude of the magnetic field produced at the ground? Compare with the Earth's magnetic field.

First, the magnitude of the field is given by

$$B = 2 \times 10^{-7} \frac{97}{9} T = 2.2 \mu T.$$

The direction is given by the right hand rule: holding your thumb in the west direction, the bottom of the fingers will point to the South. Thus the magnetic field is $2.2 \mu T$ pointing South.

The Earth's field is about 50 μT (half a Gauss) pointing to the North. (Varies a little bit). Thus this power line produces a magnetic field that is small compared to the Earth's field:

$$\frac{B}{B_{Earth}} = \frac{2.2}{50} = 4.4\%.$$

The power line diminishes the natural magnetic field at the surface by 4.4%.

2 Force Between Parallel Wires

If you have to infinitely long parallel wires, each produces a magnetic field which then exerts a force on the other. This force is attractive if the currents are in the same direction and repulsive if they are in opposite directions. (Use the right hand rule to prove this to yourself.) From the Lorentz force law and the above formula for the magnetic field of a wire, the force per unit length has magnitude

$$f = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r}.$$

Here, $I_1$and I_2 are the currents and r the distance between the wires.

3 Definition of Ampere and Coulomb

The official definition of the unit of current is based on this formula. A Coulomb is defined as the amount of charge that is transmitted by a current of 1 Ampere in one second. You also see where the value of μ_0 comes from.

> One Ampere is defined to be the constant current which will produce an attractive force of 2×10^{-7} Newtons per meter of length between two straight, parallel conductors of infinite length and negligible circular cross section placed one meter apart in free space.

A better definition of a Coulomb is as the magnitude of the charge on a certain number of electrons, chosen to agree with the old definition: $6.24150962915265 \times 10^{18}$ electrons. This is going to happen but it takes a while for the international committees to take action. This level of hair-splitting with definitions is not of interest in applications. I include it only because someone asked me about it.

4 Ampere's Law

Just as Gauss's Law is more convenient to use than Coulomb's Law, there is a way to restate the magnetic field caused by a current: Ampere's Law.

This time we relate the line integral of the magnetic field around a closed curve to the current that crosses a surface enlosed by it.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{encl}.$$

4.1 Example: Long Thin Straight Wire

Just to see that this gives the correct answer for a infinite straight wire, imagine the closed curve to be a circle around the wire of radius r. The magnetic field points along the circle at all points, so the left hand side is

$$2\pi r B$$

while the right hand side is $\mu_0 I$. So $B = \frac{\mu_0}{2\pi} \frac{I}{r}$.

4.2 Example: Long But Thick Wire

Suppose that the current is distributed uniformly in the interior of a thick wire of radius R. The magnetic field outside will be the same as if all the current is concentrated at the center of the wire.

(You can see this using Ampere's law as before.)

But inside the magnetic field will be zero at the central axis of the wire and increase towards the surface of the wire. If $r < R$, the integral around a circle of radius r is

$$\oint \mathbf{B} \cdot d\mathbf{l} = 2\pi r B(r).$$

The current density is the current divided by the cross-sectional area of the wire:

$$j = \frac{I}{\pi R^2}.$$

And the current enclosed by a circle of radius r is

$$I_{encl} = j\pi r^2 = \frac{I r^2}{R^2}.$$

Thus

$$2\pi r B(r) = \mu_0 \frac{Ir^2}{R^2}.$$

and

$$B(r) = \frac{\mu_0}{2\pi} \frac{I}{R^2} r, \quad \text{for } r < R$$

It increases to a maximum value at the surface of the wire and then decreases

$$B(r) = \frac{\mu_0}{2\pi} \frac{I}{r}, \quad \text{for} \quad r > R.$$

Note that the two formulas agree when $r = R$.

5 Electromagnets

Permanent magnets are one way of creating a magnetic field. They are convenient because we don't need a source of electricity. But what if we want to turn the magnetic field on and off? It is better then to create a magnetic field using an electric current. But single straight wires don't produce strong enough fields for currents that we can easily produce.

We need to magnify the magnetic field produced by a fixed current by winding the wire around a cylindrical shape: asolenoid. The strongest fields are then in the cylindrical cavity inside the solenoid: the field outside an infinitely long solenoid is zero. Using Ampere's law on a rectangle with one inside and another outside we get

$$B = \mu_0 n I$$

where n is the number of turns of the wire per unit length.

5.1 Example

A 56.0 cm -long solenoid 1.35 cm in diameter is to produce a field of 0.375 mT at its center. How much current should the solenoid carry if it has 755 turns of wire?

$$I = \frac{B}{\mu_0 n}$$

We have $B = 0.375 \times 10^{-3} T$, $n = \frac{755}{.56} = 1348 m^{-1}$, $\mu_0 = 4\pi \times 10^{-7} T m A^{-1}$

$$I = \frac{0.375 \times 10^{-3}}{1348 \times 4\pi \times 10^{-7}} = \frac{0.375 \times 10^{1}}{1.35 \times 4\pi} = .22A$$

5.2 Ferromagnets

Some materials (iron) are made of molecules each of which has a magnetic moment: they are little permanent magnets. At high temperatures (above the Curie temperature, which is 1043 K for iron) these are oriented in random directions so the net magnetism is zero. But if they are cooled below the Curie temperature, most of them (not all) line up the same direction. That is how permanent magnets are made. Such materials are called ferromagnets, because iron was the first such material to be discovered.

If you place a ferromagnet in a magnetic field even more of the molecules line up along the field, enhancing the field. So we can combine an electromagnet and a ferromagnet to produce an even stronger magnetic field. A solenoid with a ferromagnetic core has a magnetic field inside equal to

$$B = \mu n I$$

where the constant $\mu$is the permeability of the core. If the core is vacuum (or air) μ is equal to μ_0 the permeability of the vacuum. For iron the permeability can be from a 100 times μ_0 to 10^5 times $\mu_0$depending on the exact manufacturing method. You can see why the electromagnets often have an iron core.