

18. Electromagnetic Waves

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1 Energy Density of Electromagnetic Fields

Electric fields accelerate charged particles. The energy the particles gain must come from the field. So electric fields must carry energy. Same for momentum.

This is hard to understand at first because the electric field is something that can exist in empty space, far away from its sources. How can the energy and momentum get to such a distant point?

The answer is that it is carried by electromagnetic waves. As the waves move through space, they carry energy and momentum with them at the speed of light.

It is possible to calculate the energy of electric and magnetic fields from the physics we learned so far. Inside a capacitor there is a constant electric field. The total energy in the capacitor is $\frac{1}{2}CV^2$. The electric field is $E = \frac{V}{d}$ where d is the distance between the plates. Hence the energy per unit volume is

$$U_E = \frac{1}{2} \frac{CV^2}{Ad}$$

where A is the area of the plates. Also, $C = \epsilon_0 \frac{A}{d}$. Thus

$$U_E = \frac{1}{2} \epsilon_0 \frac{V^2}{d^2}$$

But the electric field is $E = \frac{V}{d}$. Thus the energy per unit volume of electric field is

$$U_E = \frac{1}{2} \epsilon_0 E^2.$$

By looking at the energy of an inductor, $\frac{1}{2}LI^2$ and the formulas for the magnetic field and inductance we can derive similarly the magnetic energy per unit volume

$$U_B = \frac{1}{2} \frac{B^2}{\mu_0}.$$

Thus the total energy density of an electromagnetic field is

$$U = U_E + U_B = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0}.$$

2 The Poynting Vector

As a wave moves through space it carries an energy per unit volume given by the above formula. The amount of energy crossing per unit area and unit time can be thought of as the flux of a vector, called the Poynting vector \mathbf{S} . It points in the direction in which the wave is moving. Its magnitude can be calculated, knowing the energy density and the speed of the wave. Thus we get (see the text for the argument)

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}.$$

The energy transported across a surface per unit time is

$$\int \mathbf{S} \cdot d\mathbf{A}.$$

Because \mathbf{E} and \mathbf{B} are perpendicular to each other and to the direction on propagation, this vector is pointed along the direction of propagation and has magnitude $\frac{1}{\mu_0}EB$. Using $E = cB$ and $\epsilon_0\mu_0 = \frac{1}{c^2}$, the magnitude of the Poynting vector can also be written in other ways.

$$S = \epsilon_0 c E^2 = \frac{cB^2}{\mu_0}.$$

In a wave the electric and magnetic fields vary periodically with time; so often we talk of the average energy flux, which is given by the rms electric and magnetic fields:

$$S_{av} = \epsilon_0 c E_{rms}^2 = \frac{cB_{rms}^2}{\mu_0}.$$

2.1 Example

How much energy is transported across a 7.00cm^2 area per hour by an EM wave whose E field has an rms strength of 33.0mV/m ?

Assume that the area is normal to the direction of the wave). Energy transported is

(area)* (the magnitude of the Poynting vector)*(time)

$$S_{av}AT = A\epsilon_0 c E_{rms}^2 T =$$

$$A = 7 * 10^{-4}m^2, \quad T = 3600s$$

so that the energy transported is

$$7.3\mu J.$$

3 Inverse Square Law for Energy of Wave

We know that a static point charge creates an electric field that decreases like r^{-2} . But this is not true of an antenna radiating an electromagnetic wave. The field actually decreases like r^{-1} .

To understand why, suppose the antenna is turned on and off. It produces a pulse of radiation that the energy spreads through space in a wave. In a time t it reaches a distance $r = ct$. The area of the sphere of radius r is proportional to r^2 . The total energy crossing the sphere is the same as was released by the antenna. As the sphere expands, the energy per unit area (the magnitude of the Poynting vector) must decrease like r^{-2} . Since $S \propto E^2$ the electric field must decrease like r^{-1} .

3.1 Example

A radio station is allowed to broadcast at an average power not to exceed 22 kW. If an electric field amplitude of 2.3×10^{-2} V/m is considered to be acceptable for receiving the radio transmission, estimate how many kilometers away you might be able to hear this station.

The total power crossing a sphere of radius r is

$$P = 4\pi r^2 S = 4\pi r^2 \epsilon_0 c E_{rms}^2$$

Recall that the peak electric field E_0 is related to the rms value by

$$E_{rms}^2 = \frac{1}{2} E_0^2.$$

Thus

$$P = 2\pi r^2 \epsilon_0 c E_0^2$$

Thus

$$r = \frac{1}{E} \sqrt{\frac{P}{2\pi\epsilon_0 c}}$$

which is 50km.