19. Ray Optics

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When the wave length is small light travels along straightlines called rays. Ray optics (also called geometrical optics) is the study of this light in this situation.

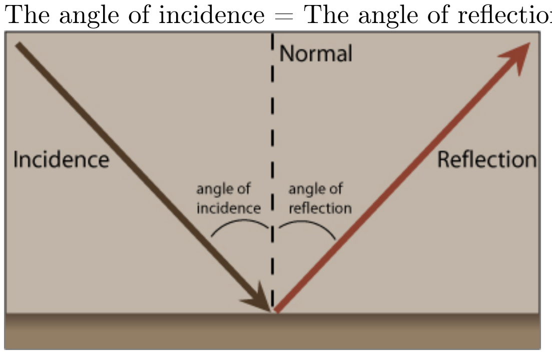
When light encounters an obstruction (a change in the medium) it can be either absorbed, reflected, or refracted (transmitted). Usually a bit of each happens: some light is absorbed, some reflected and some transmitted. A polished metal surface will reflect most of the light: mirrors work that way. Light passing from air into glass will be bent (refracted) at the interface. We will study the laws that determine the angle of reflection and refraction in each case.

By using mirrors and lenses of different shapes we can make light do all sorts of useful things: microscopes, telescopes, cameras etc. work this way.

1 The Law of Reflection

The simplest situation is light falling on a flat reflecting surface. If the light ray is normal to the surface it is reflected right back. If the ray makes an angle i with the normal, it is reflected to another ray on the other side of the normal, making an equal angle r. This is the basic law of reflection.

The angle of incidence = The angle of reflection.



1.1 Example

Using this simple principle you can show that the minimum length of a mirror than can show your whole figure is half your height. See text.

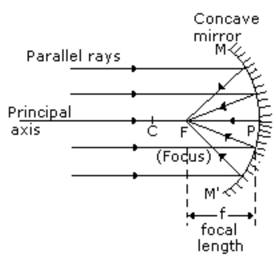
2 Concave Mirror

If the surface of the mirror is curved, different parts of a light beam will fall on it at different angles of incidence. So they will be reflected in different directions. If the mirror curves towards the light source it is said to be concave. It can be thought of as part of an arc of a circle or a piece of a sphere of a certain radius.

2.1 Parallel light beam

Suppose a beam of light from a far away source (e.g. the Sun) falls on such a mirror. All the reflected light will pass through a common point called the focus. The distance of the focus from the central point on the surface of the mirror is called the focal length f. The law of reflection implies that the focal length is half the radius of the mirror.

A technical refinement: It turns out that what I said above is only true for small mirrors. In Astronomy we need large mirrors. The shape that collects all the light to one point is not a sphere but a paraboloid. But this is beyond the scope of this course.



It is possible to light some paper on fire by concentrating sunlight with a concave mirror. Also solar power units work this way. Your satellite dish works on the same principle, except that it collects radio waves.

2.2 Image of a Source

If the source of light is at a finite distance d, the reflected light will come together at a distance i from the mirror to form an image. These distances are related by

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f}.$$

If $d = \infty$, we get $i = f$: the light collects at the focus. If $d > f$ (the source is farther than the focus) the quantity $i$that satisfies this is a positive number. This image is formed is said to be real: we can place a screen there and see the light forming a little picture of the source, like on a movie screen. This image will be inverted. It will be smaller than the original if $i < d$. The magnification of the image is $\frac{i}{d}$.

If the source is closer than the focus $d < f$ the answer for i is negative: the image is behind the mirror. In this case the image is virtual. It cannot be captured on a screen. But if you at the mirror you will see an image. It will not be inverted and will be magnified. Makeup mirrors (or shaving mirrors) work this way.

3 Convex Mirrors

Convex mirrors curve away from the source and work the opposite way from concave mirrors. A parallel beam of light is reflected into a divergent beam. It will appear as though the reflected light is coming from a point at a distance half the radius behind the mirror. We say that the focal length of a concave mirror is negative, equal in magnitude to half its radius. The distance is measured from the mirror and distance in front are positive and those behind are negative. Again, objects at a finite distance will form an image behind the mirror at a distance i given by

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f}.$$

Because f is negative and d is positive, i is always negative: the image is always virtual and is behind the mirror. The distance to the image is less than the distance to the source. Also, the image is smaller than the source.

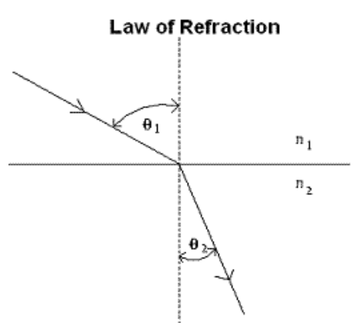
The rear view mirrors in cars are of this type. Now you know why it says

> Objects in the mirror may be closer than they appear

4 The Law of Refraction

Refraction is a more complicated business. Again, we can predict the angle of refraction (the angle the transmitted light ray makes with the normal) knowing the angle of incidence (the angle of the incident light ray with the normal). But this relation depends on a property of the material called the refractive index.

The vacuum has refractive index equal to one, by definition. So does air to a good approximation. But glass has a refractive index of about 1.5. (Varies a bit accodring to composition). Light travels slower in materials other than the vacuum. The speed of light in a material is $\frac{c}{n}$.



The law of refraction (also called Snell's Law) relates the angles of incidence θ_1 and refraction θ_2. If n_1 is the refractive index of the material on side (the incident side) and n_2 that on the other

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

If n_2 is greater (as with glass when $n_1 = 1$ for air) the angle θ_2 is smaller than θ_1 :the light is bent towards the normal.

A strange thing happens if $n_1 > n_2$ and θ_1 is big enough: there is no solution for because

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

becomes greater than one. In this case light does not pass through to the other side. Instead it is reflected back. This phenomenon is called total internal reflection. It is different from what happens at a mirror because a mirror reflects all light: here only light at certain angles is reflected.

5 Converging lenses

By using glass with a curved surface we can now focus light to a point. The laws are similar to those for mirrors. There are a couple of important differences.

A convex lens (with the surface curving away from the source) converges light.

The distance to the image is considered positive if it is on the opposite side of the source. (For a mirror positive distance for both source and image are on the same side.) Thus a real image, one that can be caught on a screen and has positive distance, will be on the opposite side of the source. (Think of a camera lens and the image on the screen). Such an image will be inverted. We still have the relation

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f}$$

with $f > 0$ for a converging lens. The magnification of the image is $\frac{i}{d}$. If the source is far away the image is small and vice versa. Note that i and d are interchangeable: there are two ways to form an image if we are allowed to move the lens.

A magnifying glass is a converging lens. So are eye glasses for far-sighted people.

6 Diverging lenses

A concave lens diverges light. A parallel beam will look like it is coming from a point on the same as the source, but at some finite distance. We regard the focal length of a divergent lens as negative. Then the relation

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f}$$

still holds.

Corrective lenses for near sighted people are divergent lenses. They will produce an image that appears to the eye to be at a closer distance than the source.

7 Combining lenses

If we place to lenses of focal lengths f_1 and f_2 right next to each other the combination can be thought of as a single lens of focal length

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}.$$

This holds even for divergent lenses if we regard its focal length is negative.

8 The eye as a lens

The eye contains a lens that focuses light onto a screen behind it, called the retina. The focal length can be changed by muscles that pull or press on the lens. The average focal length is about 2.2 cm when the eye is relaxed. In some people the lens has too small a focal length. They are near sighted. A corrective lens with a negative focal length (divergent lens) placed in front will bring the effective length back to normal. If the eye lens has too large a focal length (is not powerful enough) they are far sighted. Then a convergent lens must be used.

Bifocal eye glasses have a positive focal length in the lower portion and zero or negative focal length in the higher portion. When you read you use the lower part of the field of vision and when you look far usually use the higher part.

9 The power of a lens

Optometrists use the inverse of the focal length as a measure of the power of a lens. (Power here does not mean energy per unit time.) The unit is Diopter which is an inverse meter. Thus a lens with focal length 20cm has a power of 5 diopters. If you wear prescription glasses you might know this already. I know more than I should about this because when the Optometrists hear that I am a physicist they tell me all this, as they examine my eyes. It is usually a more pleasant conversation than I have with my dentist.

References

[1] The Snell's Law diagram is from http://hypertextbook.com/facts/2005/ShirleyDeng.shtml
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