20. Wave Optics

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In most everyday situations we can treat light as if it goes along rays. So people used to think that light was made of tiny particles. But there are some phenomena and many devices that depend on the wave properties of light When the wavelength and the size of the devices are about the same, wave phenomena like interference become important.

But the idea that light is made of particles is not entirely wrong. There are phenomena in quantum physics where light behave as if it is made of particles, each carrying an amount of energy $h\nu$ proportional to its frequency ν. This explains the photoelectric effect, for which Einstein got his Nobel Prize.

1 Wavelength Depends on Refractive Index

The frequency does not. As light passes from one medium to another, its frequency remains the same. But its wavelength changes. If the refractive index is n, the wave length in that material is $\lambda_n = \frac{\lambda}{n}$ where λ is the wavelength in vacuum.

2 The color of light depends on wavelength

Red light has a long wavelength of about 750 nm and blue light has shorter wavelength of about 450 nm. The eye cannot distinguish all the possibilities in between. It is only sensitive to three "primary colors" of wavelengths 420 nm (violet), 564 nm (yellow) and $534nm$ (green). All other colors are percieved as combinations of these.People with deficiency in one of these kinds of receptors, will have some degree of color blindness.

Most devices like computer screens are made to work with other primary colors like Red Green Blue or Red Yellow Green. Thus perceived color is not so much a physical property of light as a property of our eyes. Other species have different numbers and kinds of color receptors.

3 Interference

If two waves of the same wavelength come together, they can either reinforce each other (constructive interference) or cancel each other out (destructive interference). The first happens if the maxima of the waves coincide, in which case the minima will coincide also. In the latter case, the maximum of one happens at the minimum of the other and vice versa. Suppose $A_1$and $A_2$are both positive numbers, describing the amplitude of two waves. When the waves are "in phase" the maxima occur at the same point and the combined wave has amplitude $A_1 + A_2$:

$$A_1 \sin[kx - \omega t] + A_2 \sin[kx - \omega t] = (A_1 + A_2) \sin[kx - \omega t]$$

If the second wave has maxima displaced relative to the first, there will be a number ϕcalled the phase that describes this. When we combine

$$A_1 \sin[kx - \omega t] + A_2 \sin[kx - \omega t + \phi]$$

we can geta more complicated answer depending on ϕ. Recall that

$$\sin[\theta + \pi] = -\sin\theta.$$

So if $\phi = \pi$, we get destructive interference.

$$A_1 \sin[kx - \omega t] + A_2 \sin[kx - \omega t + \pi] = (A_1 - A_2) \sin[kx - \omega t].$$

If the waves are of equal strength $A_1 = A_2$ they will completely cancel each other! Thus we get constructive interference if ϕ is an even multiple of $\pi$and destructive interference if it is an odd multiple. A phase change of 2π has the same effect as the wave moving a full wavelength.

4 Change of phase during reflection

There is something peculiar about reflection of light from a surface of higher refractive index: the reflected light has a phase chaneg of πrelative to the transmitted light. This does not happen when light is reflected from a surface of lesser or greater refractive index. It is beyond the scope of this course to explain why this peculiar phenomenon happens. (Follows from Mawell's theory.)

5 Example: Color on Thin films

We have all seen some colores patterns on thin oil films or soap bubbles. The thickness of these films is about half a micron which is about the wavelength of light. At different angles and thicknesses light of different wavelengths will interefere constructively.

Suppose light is incident normally on oil film. Some of the light is reflected at the top layer. This reflected light will change its phase by π because oil has higher refractive index than air. Some of the light will also be reflected by the bottom layer. This light does not change its phase. To have constructive interference, so that the light from the two layers add up if

$$2t = \frac{1}{2}\frac{\lambda}{n}.$$

where t is the thickness of the film.

Light reflected at the bottom layer travels twice through the oil, so the distance it travels through oil is $2t$. Its wavelength in oil is $\frac{\lambda}{n}$. If the distance that light has travelled is a half a wavelength, it will interfere constructively because there is a phase change of π(or half a wavelength) due to reflection at the first surface.

Thus

$$t = \frac{\lambda}{4n}.$$

This is the minimum thickness. If the distance travelled is

$$2t = \frac{1}{2}\frac{\lambda}{n} + \frac{\lambda}{n}$$

again you get the same effect: a complete wavelength later the light is in the same phase.

Thus in general the condition is

$$2t = \left[m + \frac{1}{2}\right]\frac{\lambda}{n}, \quad m = 0, 1, 2, \cdots$$

6 Example: Non-Reflective Coating

We can get the opposite effect if the light is made to interfere destructively. This is how non-reflective coating on eyeglasses or camera lenses work. There is a subtle effect here though: the coating usually has a refractive index in between that of air and glass. So the light has a change of phase of π when reflected at ech interface: so this effect cancels between the two beams. The condition for destructive interference is then

$$2t = m\frac{\lambda}{2n}, \quad m = 0, 1, 2, \cdots$$

The smallest thickness is a quatrter wavelength:

$$t = \frac{\lambda}{4n}.$$

7 Newton's Rings

The wave nature of light is also evident in a famous experiment of Isaac Newton. A lens is placed on a glass plate . Light passes through the lens and is reflected from the plate then passes through the lens again to your eye (or a camera). If the little gap of air between the glass and lens is $0, \frac{1}{2}\lambda, \frac{3}{2}\lambda$, we get constructive interference- a bright ring. (In this case both light rayes are reflected at a denser material, so both are shifted by πin phase.) If the gap is $\lambda, 2\lambda$ etc. we get destructive interference: a dark ring.

Newton was fascinated by this phenomenon, which was pointed out to him by his less famous contemporary, Robert Hooke. But he did not make the correct conclusion from it: he wanted to explain everything in terms of particle mechanics. So instead of the obvious explanation in terms of waves, Newton came up with a complicated (and wrong) one in terms of particles. Occasionally, being too smart is a disadvantage.

8 Photons

But in a remarkable turnaround, about hundred years ago it was discovered that light does behave like particles in some situations. The energy of a pulse of light of frequency ν can only increase in multiples of $h\nu$ where h is a very small number called the Plank's constant. Its value is $6.63 \times 10^{-34} Js$. Thus a pulse of light can be thought of as made up of a number N of photons (the particle of light); its total energy will be

$$E = Nh\nu.$$

This number is very large (like 10^{20}) for everyday situations. So we don't notice that the energy jumps by $h\nu$; each jump is very small. But there are some effects that make it clear that this is the situation.

For example, the photoelectric effect. When light is shined on metals, electrons absorb that energy and are ejected from the metal. Each metal has a characteristic energy W (call the Work function for historical reasons) that is the amount of must be given to liberate an electron. It was found that light of too low a frequency cannot liberate electrons even if it has high intensity. Einstein's explanation was that the electrons can only absorb one photon at a time. (Actually there is a very small chance that two or more photons can be absorbed, but it is so small that it is hard to see in those experiments.) If each photon has an energy $h\nu$ the energy of the liberated electron will be

$$E = h\nu - W.$$

When $h\nu < W$ the electron cannot escape. Later experiments measured the energy of the liberated electrons and confirmed this prediction. Modern electronic cameras work on this principle.