S11 PHY114 Problem Set 1

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1 Due Monday 24 Jan 2011

1. When a plastic comb is rubbed against a cloth, $2\mu C$ of charge is transferred. How many electrons are transferred to make this happen?
2. How many electrons are there in 50 kg of water? This is roughly the number of electrons in a human body.
3. What is the attractive force between an electron and a proton in a hydrogen atom, assuming that the distance between them is 53×10^{-12}m ?
4. Two charged particles are at a distance d from each other. They are moved apart and the force is now only one quarter of the original. How far apart are they now? The charges remain the same.
5. Two charges $Q_1 = 7\mu C$ and $Q_2 = 9\mu C$ are located 1.2 m apart. Where will you place charge $Q_3 = -2\mu C$ so that it is in equilibrium? What would happen if we changed its position even slightly towards either Q_1 or Q_2 from this equilibrium position?

2 Solutions

1. Recall that the charge of an electron is $-1.6 \times 10^{-19}C$ approximately. Let n be the number of electron whose charge is equal in magnitude to $2\mu C$. Then
$$1.6 \times 10^{-19} n = 2 \times 10^{-6}.$$
$$n = \frac{2 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.3 \times 10^{13}.$$
2. Each water molecule has an oxygen atom and two hydrogen atoms. Each hydrogen atom contributes one electron. The oxygen atom has 8 electrons (this is called its atomic number). Thus each water molecule has 10 electrons. To determine how many water molecules there are in 50 kg, we need to know the average mass of a water molecule; i.e., the sum of the mass

of an oxgygen atom plus two times the mass of a hydrogen atom. We can look that up in the periodic table of elements in the inside back cover of the textbook. O has atomic weight of $16u$ and H has atomic weight of $1u$; so H_2O has atomic weight of $18u$, where u stands for atomic mass unit (amu). Also the inside front cover of the text book gives $1u = 1.7 \times 10^{-27} kg$. Thus the mass of a water molecule is $18 \times 1.7 \times 10^{-27} = 3.1 \times 10^{-26} kg$. So there are $\frac{50}{3.1\times10^{-26}} = 1.6 \times 10^{27}$ water molecules in 50 kg of water. The number of electrons in 50 kg of water is therefore 1.6×10^{28}. *Another method is to use the molar mass of water* $18 \times 10^{-3} kg/mol$ *and calculate that 50 kg of water is* $2.8 \times 10^3 mol = 1.6 \times 10^{27}$ *molecules, each with 10 electrons. So again we get* 1.6×10^{28} *electrons.*

3. We have, from Coulomb's law

$$F = k\frac{Q_1Q_2}{r^2}, \quad k = 9 \times 10^9 Nm^2C^{-2}$$

$$Q_1 = 1.6 \times 10^{-19}C = -Q_2$$

$$r = 53 \times 10^{-12}m$$

$$F = -\frac{9 \times 10^9 \left[1.6 \times 10^{-19}\right]^2}{[53 \times 10^{-12}]^2} = .0082 \times 10^{-5} = 8.2 \times 10^{-8}N$$

4. *I want the students to learn to think in terms of variables instead of numbers. Even when the final answer is a number, learn to work with variables until the last step.* By Coulomb's law the force decreases as the square of the distance. When the force is one quarter the distance must have doubled. So the answer is $2d$.

5. *This problem is meant also as a review of mechanics.* To balance the attractive force the negative charge must be placed in between the two positive charges. Let the two positive charges by at a distance d apart; and the negative charge at a distance r_1 from the charge Q_1 and at a distance r_2 from the charge Q_2. Then the condition for the total force to vanish is that they be equal in magnitude:

$$k\frac{Q_1Q_3}{r_1^2} = k\frac{Q_2Q_3}{r_2^2}, \quad r_1 + r_2 = d$$

Most factors cancel out:

$$Q_1r_2^2 = Q_2r_1^2$$

$$r_2 = \sqrt{\frac{Q_2}{Q_1}}r_1$$

$$r_1\left[1 + \sqrt{\frac{Q_2}{Q_1}}\right] = d$$

$$r_1 = \frac{d}{1+\sqrt{\frac{Q_2}{Q_1}}}$$

This is independent of Q_3. Note that if $Q_1 = Q_2$,we get $r_1 = \frac{d}{2}$: for equal charges the equilibrium is the midpoint. This is a check on our calculation. Finally, the numerical value of the distances are

$$r_1 = \frac{1.2}{1+\sqrt{\frac{9}{7}}} = 0.56m$$

$$r_2 = 1.2 - .56 = .64m$$

It makes sense that the charge Q_3 should be placed closer to Q_1 as it is smaller in magnitude: that is how we can balance the attraction from the larger charge Q_2.A small displacement towards one of the positive charges will cause a negative Q_3 to fall in: this is an unstable equilibrium.