S11 PHY114 Problem Set 2

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1 Solutions

1. The electric field at the middle point due to the charges is equal: the charges are the same in magnitude and the distances are the same and moreover, each field points in the direction from the positive to the negative charge. The field at the mid point is therefore twice that due to each charge:

$$E = 2k\frac{Q}{\left[\frac{d}{2}\right]^2}$$
$$= 8k\frac{Q}{d^2}$$

where Q is the magnitude of the charges, d the distance between them. Thus

$$Q = \frac{Ed^2}{8k}$$

Now we can put in the numbers:

$$Q = \frac{600 \times [.1]^2}{8 \times [9.0 \times 10^9]} = 8.3 \times 10^{-11}C.$$

2. Since the balls are spherical, they will produce the same electric field as a point charge placed at the center. The attractive force will be

$$F = k\frac{Q^2}{d^2}$$

where d is the distance between the balls. Thus

$$Q = d\sqrt{\frac{F}{k}}$$

in magnitude. If the charge on the electron is e in magnitude, the number of electrons transferred is

$$N = \frac{Q}{e} = \frac{d}{e}\sqrt{\frac{F}{k}}$$

$$= \frac{0.1}{1.6 \times 10^{-19}}\sqrt{\frac{1.5 \times 10^{-2}}{9.0 \times 10^{9}}}$$

$$= 0.025 \times 10^{19-\frac{11}{2}} = 0.079 \times 10^{13} = 7.9 \times 10^{11}$$

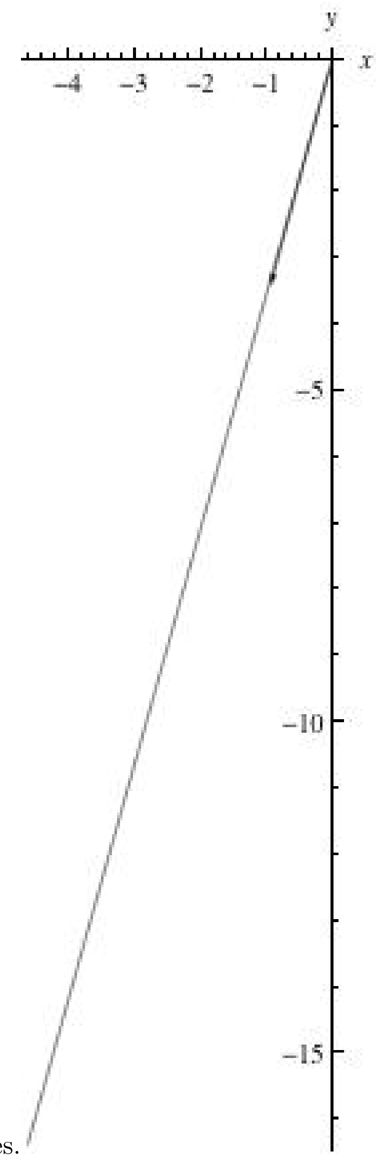
3. The force vector is the product of the charge and the electric field

$$\mathbf{F} = -4.2 \times 10^{-6}(1.1\hat{\mathbf{i}} + 3.9\hat{\mathbf{j}}) \times 10^{6} = -(4.6\hat{\mathbf{i}} + 16.4\hat{\mathbf{j}})N$$

The magnitude of this vector is $\sqrt{(4.6)^2 + (16.4)^2} = \sqrt{21.2 + 269} = 17N$. The cosine of the angle it makes with the x-axis is the ratio of the x-component to the magnitude of the vector:

$$\cos\theta = -\frac{4.6}{17} = 0.27$$

Thus the angle is either $\theta = 1.8$ radians or $2\pi - 1.8 = 4.5$ radians. (Recall that $\cos\theta = \cos[2\pi - \theta]$.)Since the y-component is also negative, we see



that it must be the larger of the two angles.

4. *This problem is also a review of mechanics.* The force on the electron is

$$\mathbf{F} = q\mathbf{E}$$

and its acceleration is

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m}.$$

The velocity at a time t is

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t = \mathbf{v}_0 + \frac{q\mathbf{E}}{m}t$$

Its position as it moves under constant acceleration is

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

The initial velocity is parallel to the electric field, so the velocity will remain parallel to it for all time. Considering only this component, the time at which it will stop is given by

$$v_0 + at = 0$$

$$t = -\frac{v_0}{a} = -\frac{mv_0}{qE}$$

Choosing as origin the initial position, the distance it travels in this time is

$$r = v_0 t + \frac{1}{2}at^2 = -at^2 + \frac{1}{2}at^2 = -\frac{1}{2}a\left[\frac{v_0}{a}\right]^2 = -\frac{v_0^2}{2a} = -\frac{mv_0^2}{2qE}$$

Putting in the numbers

$$t = -\frac{9.1 \times 10^{-31} \times 25 \times 10^6}{(-1.6 \times 10^{-19})\, 10^4} = 1.4 \times 10^{-8} s$$

and

$$r = -\frac{9.1 \times 10^{-31}\left[25 \times 10^6\right]^2}{2 \times (-1.6 \times 10^{-19})\, 10^4} = .18m$$

5. The square of the distance from the charge Q to the point x is $(x-a)^2$; the electric field at x due to this charge is $k\frac{Q}{(x-a)^2}$. Similarly, the electric field due to the charge $-Q$ at x is $-k\frac{Q}{(x+a)^2}$. Thus the total electric field at x , is pointed along the axis and has magnitude

$$k\frac{Q}{(x-a)^2} - k\frac{Q}{(x+a)^2} = kQ\left[\frac{(x+a)^2 - (x-a)^2}{(x-a)^2(x+a)^2}\right]$$

$$= kQ\frac{4ax}{(x^2-a^2)^2}.$$