S11 PHY114 Problem Set 5

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February 28, 2011

Due Monday 28 Feb 2011

1. Recall that the resitance R of a wire is proportional to its length L and inversely proportional to its cross-sectional area A; i.e., $R = \rho\frac{L}{A}$. The constant ρ is the resistivity of the material making up the wire. Copper has a resistivity of $17n\Omega m$. Household Copper wire has a diameter of $1.6mm$. Find the voltage drop across a $26m$ length of this wire carrying a $12A$ current.

2. What is the total resistance of the electrical network shown in the Figure 1?

3. Two Capacitors C_1 and $C_2$and two resistors R_1 and R_2 are connected in series. Starting from an uncharged state, how long does it take for the current to drop to half its initial value?

4. Find the equivalent resistance of an infinite ladder of resistors, each equal to 1Ω, as in the Figure 2. That is, find a single resistor connecting points A and B which is equivalent to the whole infinite ladder.

Solutions

1. The resistance is

$$R = \rho\frac{L}{\pi r^2}$$

The radius is half the diameter: $r = 0.8 \times 10^{-3}m$

$$= 17 \times 10^{-9}\frac{26}{\pi\left[0.8 \times 10^{-3}\right]^2} = 55 \times 10^{-3} = 0.22\Omega$$

Thus the voltage is

$$V = RI$$

$$= \rho\frac{L}{\pi r^2}I$$

- This comes to $0.22 \times 12 = 2.64V$

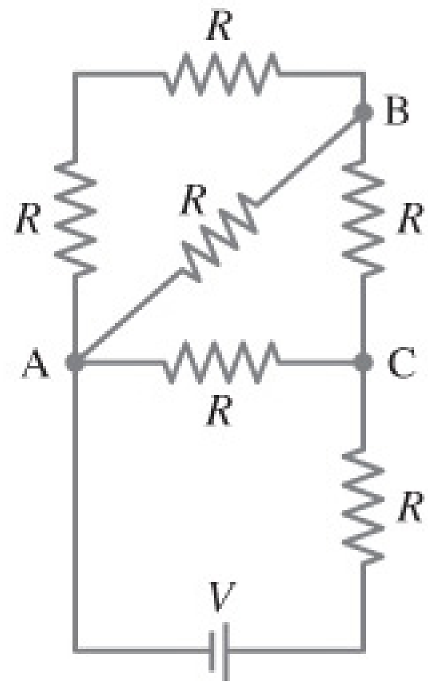

Figure 1:

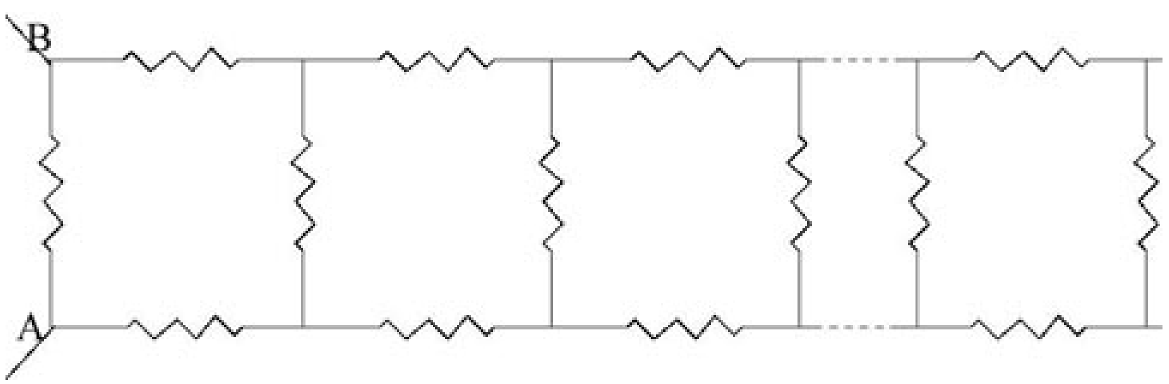

Figure 2:

2. This is a matter of using the formula for series and parallel resistances repeatedly. (Not all network problems can be solved this way though.) First of all, between points A and B there are two resistors in series and one in parallel. The two in series are equivalent to $2R$ which combined with the remaining resistor gives $\frac{(2R)R}{2R+R} = \frac{2}{3}R$. Thus we can replace those three by a single resistor $\frac{2}{3}R$ connecting A to B. This $\frac{2}{3}R$ is now in series with the resistor between B and C: combining them we get $\left(\frac{2}{3}+1\right)R = \frac{5}{3}R$. This $\frac{5}{3}R$ is now in parallel with the resistor in between A and C. Combining them we get $\frac{\frac{5}{3}R^2}{\left(\frac{5}{3}R+R\right)} = \frac{5}{8}R$. And finally, this is in series with the last resistor so that the combined resistance is $\left(\frac{5}{8}+1\right)R = \frac{13}{8}R$.

3. Two capacitors in series combine to give an equivalent capacitance $C = \frac{C_1C_2}{C_1+C_2}$.The two resistors in series combine to give an equivalent resistance $R = R_1 + R_2$. The time constant is RC. That is, the dependence of the current on time is

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

where I_0 is the maximum current, which is at the initial time $t = 0$. For the current to drop to half its initial value

$$e^{-\frac{t}{Rc}} = \frac{1}{2} \implies \frac{t}{RC} = \ln 2$$

Thus

$$t = RC \ln 2$$

$$t = \frac{C_1C_2(R_1+R_2)}{C_1+C_2} \ln 2.$$

4. Let us suppose that the equivalent resistance of the inifinite ladder is R. Then we can replace the all the resistors to the right of the first rung by a single resistor R, to get a circuit as in the figure. The resistors on the top the right side and the bottom are in series, adding to $2+R$. This is in parallel with the resistor on the left so that the combined resistance is

$$\frac{1(2+R)}{1+(2+R)} = \frac{2+R}{3+R}.$$

This must be equal to the resistance of the infinite ladder

$$R = \frac{2+R}{3+R}$$

Simplifying

$$(3+R)R = 2+R$$

$$R^2 + 2R - 2 = 0$$

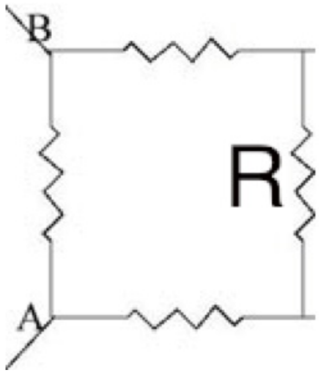

Figure 3:

$$R = \frac{-2 \pm \sqrt{2^2 + 4 \times 2}}{2} = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}\Omega$$

Clearly we must choose the positive root

$$R = \sqrt{3} - 1 \approx .73\Omega.$$