S11 PHY114 Problem Set 6

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1. A particle of mass m and charge q moves in a circular path in a magnetic field B. How does its kinetic energy depend on the radius of its orbit? Find angular momentum of a particle about the center of the circle.

2. A sort of "projectile launcher" is shown in Figure 1. A large current moves in a closed loop composed of fixed rails, a power supply, and a very light, almost frictionless bar touching the rails. A magnetic field B is perpendicular to the plane of the circuit. Should the field point up or down? If the rails are a distance d apart, and the bar has a mass of m , what constant current flow I is needed to accelerate the bar from rest to a speed v in a distance L ?

3. An atomic nucleus moves in a straight line through perpendicular electric and magnetic fields E and B respectively. If the electric field is turned off, and the magnetic field is kept the same, the particle moves in a circular path of radius r . What is the ratio of mass to charge of this nucleus?

4. A proton, a deuteron and an alpha particle enter a magnetic field B, after being accelerated from rest by the same potential difference V. The proton is found to be moving in a circle of radius R . What are the radii of the orbits of the deuteron and of the alpha particle?

5. A particle with positive charge q and mass m travels in a uniform magnetic field $\mathbf{B} = B_0\mathbf{k}$. At time $t = 0$, the particle's speed is v_0 and its velocity vector lies in the plane directed at an angle of $30°$ with respect to the y-axis as shown in the figure. At a later time t_1, the particle will cross the axis at $x = \alpha$. In terms of q, m, v_0 and B_0 , determine α and t_1.

Solutions

1. Since $\frac{mv^2}{r} = qvB$ we have $mv = qBr$ and $\frac{1}{2}mv^2 = \frac{q^2B^2}{2m}r^2$. Thus the kinetic energy is proportional to r^2. The angular momentum is $mvr = qBr^2$ so it also goes like r^2.

2. The force has magnitude $F = IdB$. The right hand rule shows that the field must point down. The work done by this force over a distance L is FL.
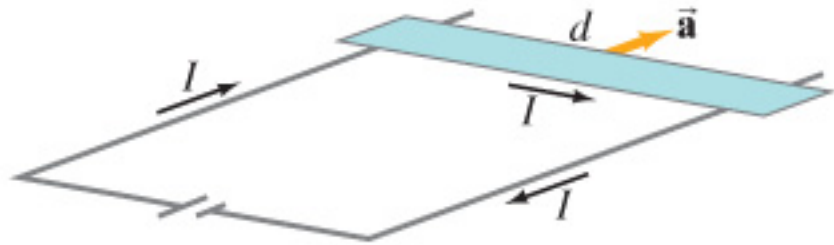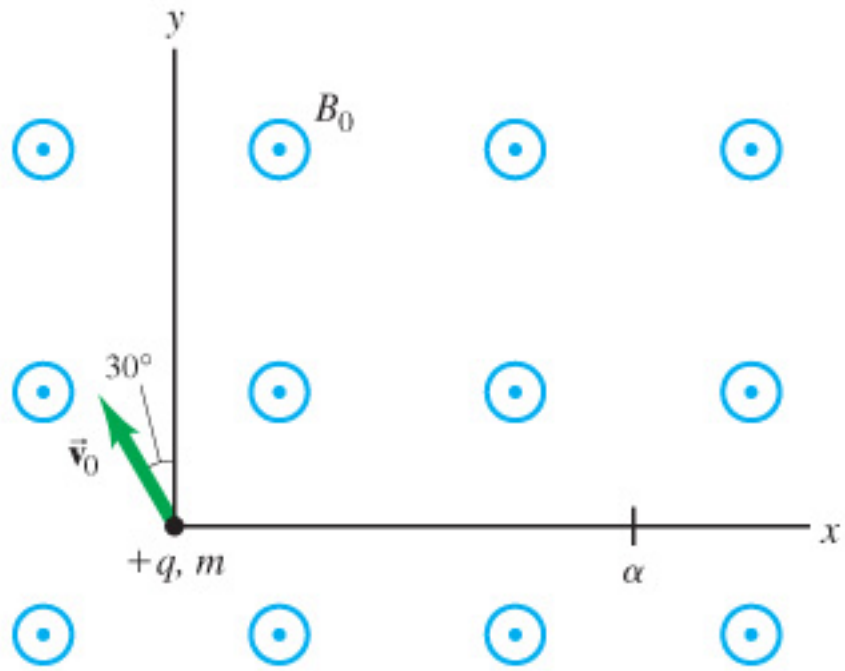
Figure 1:

Figure 2:

This must be equal to the final kinetic energy of the bar

$$\frac{1}{2}mv^2 = FL$$

Thus

$$\frac{1}{2}mv^2 = IdBL$$

$$I = \frac{mv^2}{2BdL}.$$

3. If the particle is moving in a straightline the electric and magnetic forces cancel each other. For this to happen, the two fields must be perpendicular to each other and to the velocity of the particle. Moreover the magnitudes of the forces must be equal:

$$qE = qvB \implies v = \frac{E}{B}$$

The speed remains the same once the electric field is turned off and the particle is in circular motion.

$$r = \frac{mv}{qB} = \frac{mE}{qB^2}$$

Thus

$$\frac{m}{q} = \frac{B^2 r}{E}$$

4. The radius of the orbit of a charged particle in a magnetic field is given by

$$r = \frac{mv}{qB}$$

Since the particle was accelerated from rest through a potential V

$$\frac{1}{2}mv^2 = qV$$

so that

$$v = \sqrt{\frac{2qV}{m}}$$

Thus

$$r = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \sqrt{\frac{m}{q}}\frac{\sqrt{2V}}{B}$$

Since V and B are the same for all the particles, we just need to know $\frac{m}{q}$ for each of them.

Now, the deuteron has mass that is twice the mass of the proton and the same charge so for it $\frac{m}{q}$ is twice that of the proton. Its radius is $\sqrt{2}R$.

The alpha particle has twice the charge and four times the mass so it also has $\frac{m}{q}$ equal to twice that of the proton. Thus again $\sqrt{2}R$ is the radius of its orbit.

5. We know the initial velocity and position; we know the charge, mass and magnetic field. We have to find where and when the particle will cross the x-axis. Recall that a charged particle in a magnetic field has uniform circular motion. We need to find the center and radius of this circle.

Recall that

$$m\frac{v_0^2}{r} = qv_0B_0$$

so that

$$r = \frac{mv_0}{qB_0}$$

is the distance from the initial position of the particle to the center. The direction of the vector connecting the particle to the center is perpendicular to the velocity. Since the initial velocity makes an angle of $30°$ with the y-axis, the vector towards the center makes an angle of $30°$ with the x-axis. Thus the center, the origin and the pont α form an isosceles triangle with two angles equal to $30°$ and two sides equal to r. It follows that

$$\frac{\alpha}{2} = r\cos 30°$$

$$\alpha = 2\cos 30° r = \sqrt{3}r$$

$$\alpha = \sqrt{3}\frac{mv_0}{qB_0}$$

We know the angular velocity ω of the particle to be a constant

$$\omega = \frac{v_0}{r}$$

$$= \frac{qB_0}{m}$$

The time it takes for the particle to arrive at α will be the angle it turned divided by ω. The angle at the apex of the isosceles triangle mentioned above is $180 - 2 \times 30 = 120$degrees. The angle the particle turned is $360 - 120 = 240$degrees which is $\frac{\pi}{180}240 = \frac{4}{3}\pi$ radians. Thus

$$t_1 = \frac{\frac{4}{3}\pi}{\omega}$$

$$t_1 = \frac{4}{3}\pi\frac{m}{qB_0}$$

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