S11 PHY114 Problem Set 7

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Due Monday 28 Mar 2011

1. An iron atom has a magnetic dipole moment of about $1.8 \times 10^{-23} Am^2$. Determine the dipole moment of an iron bar 9.5cm long 1.6 cm wide and 2.0 cm thick, assuming that all the atomic dipole moments are pointing along its length (i..e, it is saturated.) What torque would be exerted on this bar if is placed in a magnetic field of $0.8T$, acting at right angles to the bar?

2. A simple generator has a square coil of side a with Nloops. How many times must it turn in a magnetic field B second to produce a peak output emf of V volts?

3. The primary windings of a transformer which has an 88% efficiency are connected to a 120V ac. The secondary windings are connected across a 3Ω, $75W$ light bulb. Calculate the current through the primary coil of the transformer.Calculate the ratio of the number of primary windings of the transformer to the number of secondary windings of the transformer.

4. Suppose we apply an external voltage $V(t) = V_0 \sin \omega t$ to a circuit with resistance $R$and capacitance C. Then the equation relating the charge, its time derivative and the external voltage, is

$$R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \sin \omega t$$

- Determine constants A and ϕ in terms of V_0, R, C, ω so that the following is a solution:
$$Q(t) = A \sin[\omega t + \phi]$$
- What is the current as a function of time?
- Determine the peak current I_0 as a function of V_0, R, C and ω.
- Show that the peak current grows as a function of ω. This is an example of a "high-pass" filter which reduces low frequency inputs.

Hint You will find the following identities useful

$$\sin[\theta + \phi] = \sin \theta \cos \phi + \sin \phi \cos \theta, \quad \cos[\theta + \phi] = \cos \theta \cos \phi - \sin \theta \sin \phi$$

$$\cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$$

5. Recall that the energy of a coil (solenoid) carrying a current I and with inductance L is $\frac{1}{2}LI^2$. Find a formula for the energy density of a magnetic field by considering a coil of cross-sectional area A and length $l >> r$ with N turns. Compare with the energy density of an electric field. Typical large electric and magnetic fields that can be produced in the laboratory are about $10^4 V/m$ and $1T$ respectively. Which of these fields carries the greater energy density?

Solutions

1. We need to know how many iron atoms are in the bar. For this we need the mass of an iron atom and the density of iron: together they will tell us how many iron atoms are there in one cubic meter. The density of iron is $7.87g/c.c. = 7.87 \times 10^{-3} \times (10^2)^3 kg\ m^{-3} = 7.87 \times 10^3 kg\ m^{-3}$. An iron atom has an atomic mass of $55.85amu = 55.85 \times 1.66 \times 10^{-27} kg$. Thus there are $\frac{7.87\times10^3}{55.85\times1.66\times10^{-27}} = 8.49 \times 10^{28}$ atoms in 1 m^3 of iron. The volume of our bar of iron is $.095 \times .016 \times .02 = 3.0 \times 10^{-5} m^3$, so it contains 25.45×10^{23}atoms. The total magnetic moment at saturation is $25.45 \times 1.8A\ m^2 = 45.8A\ m^2$.

At right angles the torque is the just the product of magnetic moment and the field: $45.8 \times .8 = 36.7N\ m$.

2. The flux is $\Phi(t) = Ba^2N\cos\theta$ where θ is the angle that the normal of the plane of the coil makes with the magnetic field. If it is rotating $f$times a second,

$$\theta(t) = 2\pi f t$$

Thus the emf produced is

$$-\frac{d\Phi}{dt} = Ba^2N2\pi f\sin[2\pi f t]$$

Thus the peak emf is

$$V = Ba^2N2\pi f$$

Or

$$f = \frac{V}{2\pi a^2 BN}$$

3. Since 88%of the power is lost, we see that the power of the ac input to the primary is $P_p = \frac{75}{.88} = 85.2W$. This means that the rms current in the primary is

$$I_p = \frac{P_p}{V_p} = \frac{85.2}{120} = .71A$$

From the resistance and the power of the buld we can find the voltage in the secondary circuit

$$P_s = \frac{V_s^2}{R_s}$$

$$V_s = \sqrt{R_s P_s} = \sqrt{75 \times 3} = 15V$$

Thus we have a step down transformer that converts $120V$ to 15V. The ratio of turns is

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{120}{15} = 8$$

4. If we have

$$R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \sin \omega t$$

and

$$Q = A \sin[\omega t + \phi]$$

we get

$$AR\omega \cos[\omega t + \phi] + \frac{A}{C} \sin[\omega t + \phi] = V_0 \sin \omega t$$

But we have

$$\sin[\theta + \phi] = \sin\theta \cos\phi + \sin\phi \cos\theta$$

$$\cos[\theta + \phi] = \cos\theta \cos\phi - \sin\theta \sin\phi$$

So with $\theta = \omega t$

$$AR\omega \left[\cos\theta \cos\phi - \sin\theta \sin\phi\right] + \frac{A}{C}\left[\sin\theta \cos\phi + \sin\phi \cos\theta\right] = V_0 \sin\theta$$

Since there is no $\cos\theta$ on the r.h.s., the terms involving it must cancel:

$$AR\omega \cos\phi + \frac{A}{C} \sin\phi = 0, \implies \tan\phi = -\omega RC.$$

So

$$AR\omega \left[-\sin\theta \sin\phi\right] + \frac{A}{C}\left[\sin\theta \cos\phi\right] = V_0 \sin\theta$$

gives

$$\left[-R\omega \sin\phi + \frac{1}{C}\cos\phi\right] A = V_0$$

$$[-RC\omega \sin\phi + \cos\phi]\, A = CV_0$$

$$[\tan\phi \sin\phi + \cos\phi]\, A = CV_0$$

$$\left[\sin^2\phi + \cos^2\phi\right] A = \cos\phi C V_0$$

$$A = V_0 C \cos\phi$$

But

$$\cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$$

so that

$$A = \frac{V_0 C}{\sqrt{1+\omega^2 R^2 C^2}}$$

The current is the derivative of the charge,

$$Q(t) = A \sin[\omega t + \phi]$$

$$I(t) = A\omega \cos[\omega t + \phi]$$

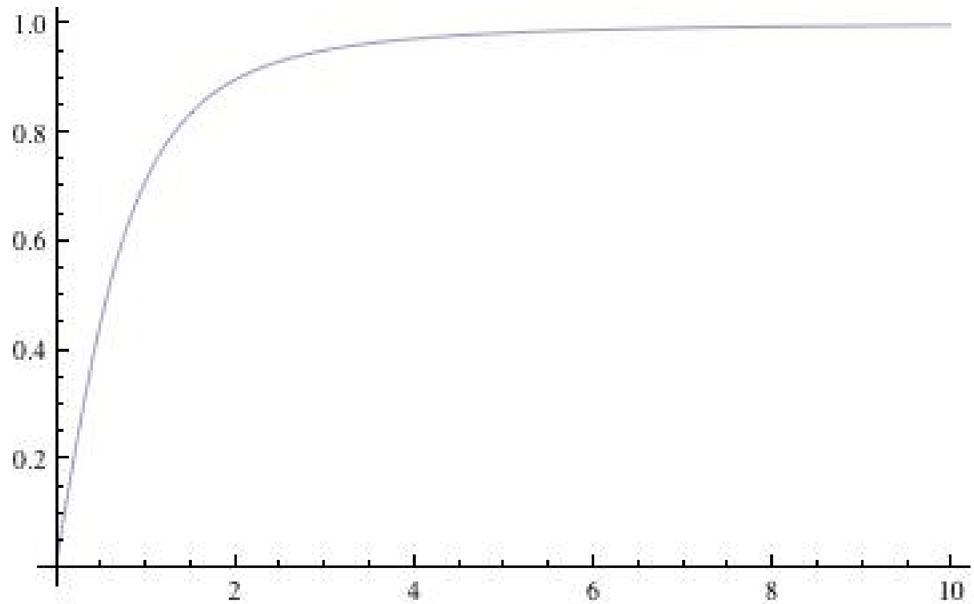
The peak current is

$$I_0 = A\omega$$

or,

$$I_0 = V_0 C \frac{\omega}{\sqrt{1+R^2C^2\omega^2}}$$

For $\omega = 0$ this is zero. It grows to $\frac{V_0}{R}$ as $\omega \to \infty$.



5. From Ampere's law we can deduce that the magnetic field of a coil (solenoid) is

$$B = \mu_0 I \frac{N}{l}.$$

On the other hand the inductance of a coil follows from Faraday's law

$$L = \frac{\mu_0 N^2 A}{l}$$

Thus the energy in a solenoid is

$$\frac{1}{2}\frac{\mu_0 N^2 A}{l} I^2$$

But

$$I = \frac{Bl}{\mu_0 N}$$

so that the energy is

$$\frac{1}{2}\frac{\mu_0 N^2 A}{l}\left[\frac{Bl}{\mu_0 N}\right]^2 = \frac{1}{2}\frac{B^2}{\mu_0}[Al]$$

The last factor is the volume inside the solenoid. So a magnetic field has energy density

$$\frac{1}{2}\frac{B^2}{\mu_0}.$$

We derived in an earlier problem that the energy density of an electric field is

$$\frac{1}{2}\epsilon_0 E^2.$$

For a field of $10^4 V/m$ the energy density is

$$\frac{1}{2}8.8 \times 10^{-12} \times 10^8 = 4.4 \times 10^{-4} J\ m^{-3}.$$

A magnetic field of one Tesla carries an energy density

$$\frac{1}{2}\frac{1}{4\pi \times 10^{-7}} = 4.0 \times 10^5 J\ m^{-3}.$$

Thus the magnetic fields in labs usually carry a lot more energy. But this energy is lost more easily as the currents encounter resistance. So such large magnetic fields are usually produced using superconducting coils.