S11 PHY114 Problem Set 8

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1.

- Find the wavelength of the radio waves emitted by the University of Rochester radio station (88.5 Mhz)?
- The number of bits per second transmitted on a radio signal is about ten times less than the frequency of the wave. A wireless local area network (WLAN) requires about 100 Mbits/sec. What frequency and wavelength of radio waves are needed?
- Visible light has wavelength in the range of 380-750nm (violet to red). Find the corresponding frequencies.
- The Voyager 1 spacecraft is now at a distance of 117 AU (astronomical unit-the distance between the Earth and the Sun). If you send a signal to Voyager 1, how long would you have to wait for a response?

2. A geosynchronous satellite orbits at the same as rate as the Earth's rotation, so that to an observer on the Earth it will appear stationary in sky. At what latitude should this satellite be located in the sky? At what angle to the horizontal should a satellite antenna at Rochester have to be pointed, assuming that the longitude of the satellite is the same as ours? At what latitude would communication with a geosynchronous satellite become impossible?What is the time delay in a telephone signal which is bounced off a satellite in geoscynchronous orbit ? A quarter-second delay would be noticeable.

3.

- Estimate the average optical power output of the Sun, knowing that about 1000 Watts per square meter reaches the Earth.
- A radio telescope can detect signals as low as $10^{-23}Wm^{-2}$. Assuming that the maximum power of an alien radio transmitter is about the same as that of a star, what is the maximum distance from which we can expect to recieve a signal?

- The United States consumes about $30 \times 10^{15} Wh$ (Watt-hours) of energy in a year. How large a field of solar panels would it take to power this usage? Assume a 10% efficiency for solar panels.

4. Calculate the displacement current between the square plates of side $9.0cm$, of a capacitor if the electric field is changing at a rate of $1.8 \times 10^6 V(ms)^{-1}$.

Solutions

1. The wavelength of a radio wave with frequency 88.5MHz is,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 ms^{-1}}{88.5 \times 10^6 s^{-1}} \approx 3.4m$$

To transmit 100Mbits/sec we need a frequency of roughly 1GHz which corresponds to a wavelength of $\lambda = \frac{3 \times 10^8}{10^9} \approx 0.3m$

The frequency of light of wavelength 380nm (violet) is

$$f = \frac{3 \times 10^8}{380 \times 10^{-9}} \approx 7.8 \times 10^{14} Hz$$

and of 750 nm (red) is

$$f = \frac{3 \times 10^8}{750 \times 10^{-9}} \approx 4.0 \times 10^{14} Hz$$

An AU is about 150 million km or about $150 \times 10^9 m$. The round trip distance to the Voyager is 234AU$\approx 35 \times 10^{12}$m The time it takes for light to travel this distance is $\frac{35 \times 10^{12}}{3 \times 10^8} \approx 12 \times 10^4 s \approx 33$ hours. So it takes more than a day to get a response from Voyager 1.

2. The radius of orbit of a geosynchronous satellite is determined by Newtonian mechanics. Equating centripetal acceleration to the gravitational acceleration,

$$\frac{GM_E}{r^2} = r\omega^2, \quad \omega = \frac{2\pi}{T}$$

$$r^3 = \frac{GM_E}{4\pi^2} T^2$$

The gravitational acceleration at the surface of the Earth is

$$g_E = \frac{GM_E}{R_E^2}$$

$$r^3 = \frac{g_E {R_E}^2}{4\pi^2} T^2$$

This basically Kepler's third law.

$$r = \left[\frac{g_E {R_E}^2}{4\pi^2}\right]^{\frac{1}{3}} T^{\frac{2}{3}}$$

Since

$$g_E \approx 9.8ms^{-2}, R_E \approx 6.4 \times 10^6 m$$

for a period of orbit of one day,

$$T = 24 \times 60 \times 60 = 86.4 \times 10^3 s$$

we get

$$r = \left[\frac{9.8 \times \left(6.4 \times 10^6\right)^2}{4\pi^2}\right]^{\frac{1}{3}} \left(86.4 \times 10^3\right)^{\frac{2}{3}}$$

$$= \left[\frac{9.8 \times (6.4)^2 \times 86.4}{4\pi^2}\right]^{\frac{1}{3}} 10^6 m$$

$$\approx 42 \times 10^3 km$$

This orbit must be over the equator in order that the angular velocity of the satellite be oriented along the angular velocity of the Earth. Thus from Rochester antennas have to point south to see a satellite over the Equator at our longitude.

The angle χ to the horizontal is given by some trigonometry. Imagine a triangle with vertices at the center of the Earth (O), a point (P) on the Earth and the satellite (Q). The angle POQ is the latitude θ. The angle OPQ= ϕ is 90° plus the angle that the line of sight PQ makes with the horizontal at P.

$$\phi = 90^\circ + \chi.$$

The side OP is the radius of the Earth R_E.The side OQ is the radius of the satellite's orbit. By the law of sines of trigonometry

$$\frac{OQ}{\sin OPQ} = \frac{OP}{\sin OQP}.$$

But

$$OQP = 180^\circ - (OPQ + POQ) = 180^\circ - (\theta + \phi).$$

Thus

$$\sin OQP = \sin[\theta + \phi].$$

$$\frac{r}{\sin\phi} = \frac{R_E}{\sin[\theta+\phi]}$$

Thus ϕ is determined as the solution to the equation

$$\sin\phi = \frac{r}{R_E}\sin[\theta+\phi]$$

That is

$$\sin\phi = \frac{r}{R_E}\left\{\sin\theta\cos\phi + \cos\theta\sin\phi\right\}$$

$$\frac{R_E}{r} = \sin\theta\cot\phi + \cos\theta$$

$$\cot\phi = \frac{\frac{R_E}{r} - \cos\theta}{\sin\theta}.$$

$$\cot[90^\circ + \chi] = \frac{\frac{R_E}{r} - \cos\theta}{\sin\theta}$$

$$\tan\chi = \frac{\cos\theta - \frac{R_E}{r}}{\sin\theta}$$

Since

$$\frac{R_E}{r} \approx 0.15, \quad \theta \approx 43^\circ$$

$$\tan\chi = \frac{0.73 - 0.15}{0.68} = 0.85$$

$$\chi = 40^\circ$$

At a latitude of $\arccos[\frac{R_E}{r}] \approx 81^\circ$ the satellite antenna would have to be horizontal to see the satellite; at any higher latitude it becomes impossible, as the line of sight to the satellite is blocked by the Earth. In practice, because of hills and buildings, communication becomes difficult above about sixty degrees latitude.

The distance to the satellite is given by the cosine formula

$$d = \sqrt{R_E^2 + r^2 - 2R_E r\cos\phi} = r\left[1 - \frac{2R_E}{r}\cos\phi + \frac{R_E^2}{r^2}\right]^{\frac{1}{2}}$$

Since $\frac{R_E}{r} \approx 0.15$ we can approximate

$$d \approx r$$

and the delay is

$$\frac{2r}{c} \approx \frac{2 \times 42 \times 10^6}{3 \times 10^8} = 0.28s$$

which is noticeable.

3. The radius of the Earth's orbit is $R \approx 150 \times 10^9 m$. Hence the power output P of the Sun is at least $4\pi R^2$ times the power per unit area of sunlight at the Earth.

$$P = 4\pi \left[150 \times 10^9\right]^2 1500$$

$$\sim 4 \times 10^{26} W$$

The Sun is a typical star. If the aliens are able to somehow transmit as much power as a star, at a distance r the power recieved per square meter will be

$$\frac{1500}{r^2} R^2$$

Setting this equal to the power that can be detected by a radio telescope

$$p = \frac{1500}{r^2} R^2$$

$$r = R\sqrt{\frac{1500}{p}} = 150 \times 10^9 \sqrt{\frac{1500}{10^{-23}}} \approx 2 \times 10^{21} m \approx 2 \times 10^5 ly$$

The size of our galaxy is comparable, at a hundred thousand light years. So we should be able to detect such transmission from within the galaxy.

Converting from Watt-hour per year to Watts, the power consumption of the US is

$$3 \times 10^{16} \frac{1}{24 \times 365} = 3.4 \times 10^{12} W$$

At 10% efficiency we would need an optical power of $34 TW$ to generate enough power. Assuming a square of side L for the solar panel, this gives

$$1500 L^2 = 3.4 \times 10^{12}$$

$$L = 160 km$$

Thus a square solar panel field of side a hundred miles can power the whole United States during the day. The cost is prohibitive currently, though.

4. The displacement current is, for a capacitor of side L

$$I_D \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 L^2 \frac{dE}{dt}$$

$$I_D = 8.9 \times 10^{-12} (0.09)^2 1.8 \times 10^6$$

$$= 13 \times 10^{-8} A$$

This is too small to be oberved directly.