S11 PHY114 Problem Set 9

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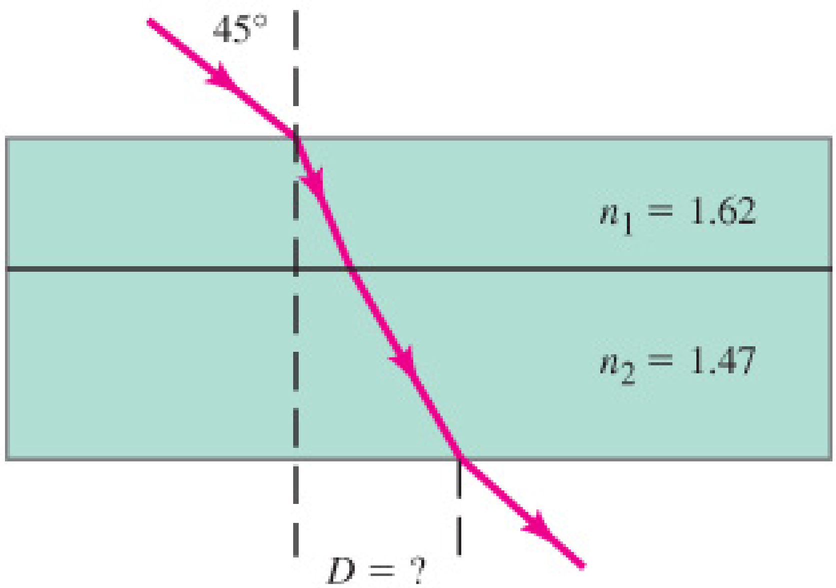
April 19, 2011

Due Monday 18 Apr 2011

1. A lighted candle is placed a distance d in front of a converging lens of focal length f_1, which in turn is a distance L in front of another converging lens of focal length f_2 . Assume that $d > f_1$. and $L > f_1 + f_2$ Calculate the position of the final image.

2. How far from a 80.0mm -focal-length lens must an object be placed if its image is to be magnified 4 times and be real? What if the image is to be virtual and magnified 4 times ?

3.A light beam strikes a 1.7 cm -thick piece of plastic with a refractive index of 1.62 at a 45 degree angle. The plastic is on top of a 2.7 cm-thick piece of glass for which the refractive index is 1.47. What is the distance D in the



figure?

4. Consider a ray of sunlight incident from air on a spherical raindrop of radius and index of refraction . Defining θ to be its incident angle, the ray then follows the path shown in the figure , exiting the drop at a "scattering

angle" ϕ compared with its original incoming direction. Find a formula for the scattering angle as a function of the incident angle. Plot this is function in the range $0 \leq \theta \leq 90^\circ$. The refractive index of water is 1.33.

Solutions

1. Let i_1 be the distance of the image formed by the first lens from it. Then

$$\frac{1}{i_1} + \frac{1}{d} = \frac{1}{f_1}, \implies i_1 = \frac{f_1 d}{d - f_1}.$$

This is positive, so it is a real image, which then serves as the object for the second lens. The distance of this object from the second lens is $d_2 = L - i_1 = L - \frac{f_1 d}{d-f_1} = \frac{Ld - Lf_1 - df_1}{d-f_1}$. The magnification is $m_1 = \frac{i_1}{d} = \frac{Ld-Lf_1-df_1}{(d-f_1)d}$

The distance i_2 of the second image from the second lens is given by

$$\begin{aligned} i_2 &= \frac{f_2 d_2}{d_2 - f_2} \\ &= f_2 \left[\frac{L - \frac{f_1 d}{d-f_1}}{L - \frac{f_1 d}{d-f_1} - f_2} \right] \\ &= f_2 \frac{Ld - Lf_1 - df_1}{Ld - Lf_1 - df_1 - f_2 d + f_1 f_2}. \end{aligned}$$

2. $m = \frac{i}{d} \implies i = md$. Then

$$\frac{1}{i} + \frac{1}{d} = \frac{1}{f} \implies \frac{1}{md} + \frac{1}{d} = \frac{1}{f}$$

$$d = f\left[1 + \frac{1}{m}\right]$$

In our case $m = 4, f = .08$ so that d=0.1 meter.

If the image is virtual, then i is negative. Then we can use the same formula to find the distance to the object for a virtual of magnification 4, but with $m = -4$

$$d = 0.08 \times \frac{3}{4} = .06 \text{ meters}$$

3. Let θ_1 be the angle that the light ray makes with the normal inside the first interface. Then

$$\sin 45^\circ = n_1 \sin\theta_1 \implies n_1 \sin\theta_1 = \frac{1}{\sqrt{2}}.$$

$$\sin\theta_1 = \frac{1}{1.62\sqrt{2}}, \quad \theta_1 = 25.5^\circ$$

Let the thickness be t_1 and d_1 the distance that the light ray moves horizontally within the first refractor. Then

$$\frac{d_1}{t_1} = \tan\theta_1, \quad d_1 = t_1 \tan\theta_1.$$

At the second interface, the incident angle is θ_1. Again

$$n_2 \sin\theta_2 = n_1 \sin\theta_1 = \frac{1}{\sqrt{2}}.$$

$$\sin\theta_2 = \frac{1}{1.47\sqrt{2}}, \quad \theta_2 = 28.8^\circ$$

$$d_2 = t_2 \tan\theta_2.$$

The total distance that the ray moves is

$$D = d_1 + d_2 = t_1 \tan\theta_1 + t_2 \tan\theta_2$$

$$= 0.81 + 1.48 = 2.29 \text{ cm}$$

4. Let θ_2 be the angle of refraction at the point that the light ray enters the water drop. This will also be the angle of incidence at the point where it is reflected (these are angles at equal sides of an isosceles triangle), which then is the angle of reflection and also the angle of incidence at the point where light leaves water (isosceles triangle again).

$$\sin\theta = n \sin\theta_2.$$

$$\theta_2 = \arcsin\left[\frac{\sin\theta}{n}\right]$$

Now, each time light is refracted, it is deflected by an angle equal to the difference between the incidence angle and the refraction angle. Each time it is reflected, the direction changes by 180° minus twice the angle of incidence. Taking into account of the two refractions and one reflection

$$\phi = 2(\theta - \theta_2) + 180^\circ - 2\theta_2.$$

That is

$$\phi = 180^\circ + 2\theta - 4\text{arcsin}\left[\frac{\sin\theta}{n}\right]$$

This is plotted in the figure.

This explains how a rainbow is formed.

When sunlight is reflected from water, the most intense light is formed when the scattering angle is a minimum: at that point small variations in the incident angle do not affect the outgoing direction, so light can accumulate in intensity. This is a scattering angle of 140 degrees according to the above graph. If the sun is behind us, and near the horizon, this is about 40 degrees from the horizontal.

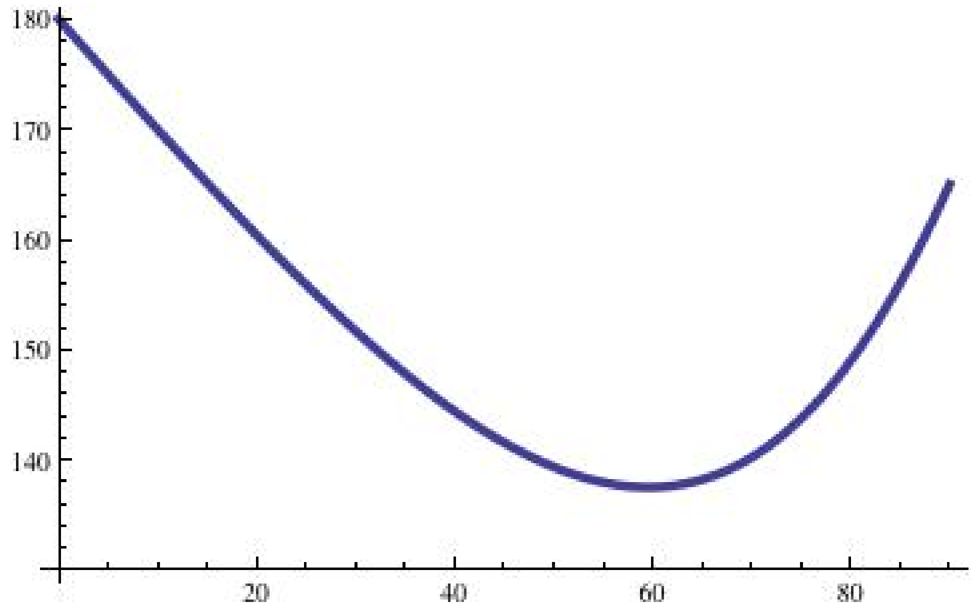


Figure 1:

Because the refractive index depends on wavelength, and wavelength determines color, light of different colors are reflected through slightly different angles: but all close to 140 degrees. This is what we see as a rainbow.