S11 PHY114 Problem Set 10

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1. A radio telescope, whose two antennas are separated by L , is designed to receive radio waves of frequency f produced by astronomical objects. The received radio waves create electronic signals in the telescope's left and right antennas. These signals then travel by equal-length cables to a centrally located amplifier, where they are added together. The telescope can be "pointed" to a certain region of the sky by adding the instantaneous signal from the right antenna to a "time-delayed" signal received by the left antenna a time ago. (This time delay of the left signal can be easily accomplished with the proper electronic circuit.) Find the time delay if the object t be viewed is at an angle θ to the vertical.

2. Stealth aircraft are designed to not reflect radar, whose wavelength is typically 2 cm , by using an antireflecting coating. Ignoring any change in wavelength in the coating, estimate its thickness.

3. A radar speed detector works by reflecting microwaves off a car and measuring the change in the frequency. If waves of frequency $4 \times 10^{10} Hz$ are measured to be 2×10^3Hz lower in frequency after reflection, what is the speed of the car? Is it moving away or towards the speed detector?

4. An X-Ray photon has an energy of 124 keV. What is its wavelength?

5. A radioactive isotope has half-life of 9 months. After three years what fraction of this isotope will be left?

6. A 3 MeV gamma ray is emitted by a lead nucleus. By how much is the mass of the nucleus reduced?

Solutions

1. The delay must compensate for the extra distance travelled by one of the signals. By drawing a triangle with two vertices at the telescopes and one side pointed towards the object we see that this extra distance is $L \sin\theta$. Thus the time delay is

$$\delta t = \frac{L \sin\theta}{c}.$$

2. The difference in path lengths between the waves reflected at the top and bottom of the coating is twice the thickness. This must be equal to half the wavelength. Thus the thickness is a quarter of the wavelength, $\frac{2}{4} = 0.5$cm.

3. Lorentz transformations are not needed since the emitted and reflected frequencies are measured in the same reference frame. They do come into play if we were to ask what is the frequency seen by the car. For a more detailed derivation than in the textbook see "Theory of Relativity" by W. Pauli (Pergamon NY 1958) pages 94-97.

The Doppler formula for the frequency of the reflected wave is

$$f_r = f \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \approx f \left(1 + 2\frac{v}{c}\right).$$

Thus

$$f_r - f = 2\frac{v}{c} f$$

$$v = \frac{f_r - f}{2f} c$$

In our case $f_r < f$ so the car is moving away. The speed is the magnitude of the above

$$|v| = \frac{2 \times 10^3}{4 \times 10^{10}} 3 \times 10^8 = 15 ms^{-1}. = 54\ km/hr$$

4. We have $E = hf = \frac{hc}{\lambda}$ so that $\lambda = \frac{hc}{E} = \frac{4 \times 10^{-15} \times 3 \times 10^8}{E}$ when E is in MeV and λ is in meters. This means that with $E = 1.24 \times 10^5 eV$ $\lambda = 10\ pm$.

5. The fraction left decreases exponetially with time. If T is the half life, after time t the fraction of the isotope left is $2^{-\frac{t}{T}}$.In our case $\frac{t}{T} = \frac{3}{\frac{9}{12}} = 4$. Thus $2^{-4} = \frac{1}{16}$ of the isotope is left.

6. We use $E = mc^2$ so that

$$m = \frac{E}{c^2} = \frac{3 \times 10^6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2} = 5.3 \times 10^{-30} kg$$