Electric flux, Gauss’ s law

Physics 122

Concepts

• Primary concepts:
  Electric flux

Laws

• Gauss’s law
Definition of electric flux

- Flux means flow
- Nothing is actually flowing
- Other than that thinking of flow really helps to understand the flux

Electric flux

- If electric field is rain – electric flux is the amount of water in a bucket accumulated per unit of time:

\[ \Phi = E \cdot A = E A \cos \theta \]

Only component of the field perpendicular to the area \( A \) contributes to the flux.

Electric flux

- Alternatively we can define a vector \( \vec{A} \), which equals to the area \( A \) and is directed perpendicularly to the area.

\[ \Phi = \vec{E} \cdot \vec{A} = E A \cos \theta \]

Flux is proportional to the number of field lines going through the surface.
Compare fluxes

Which flux is larger?

1). $\Phi_1 > \Phi_2$
2). $\Phi_1 < \Phi_2$
3). $\Phi_1 = \Phi_2$

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Compare fluxes

Which flux is larger?

1). \( \Phi_1 > \Phi_2 \)  
2). \( \Phi_1 < \Phi_2 \)  
3). \( \Phi_1 = \Phi_2 \)
Non-uniform field, irregular surface

\[ \Phi = \sum_{i} E_i \cdot dA_i \]
\[ \Phi = \int \vec{E} \cdot dA \]

Closed surface

\[ \Phi = \int \vec{E} \cdot dA \]

Influx < 0

Outflux > 0

Don’t worry, we’ll only deal with simple surfaces. Complex surfaces can be handled using numeric integration by a computer.

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Why do we need flux?

Gauss’s law:

\[ \Phi = \frac{Q_{net}}{\varepsilon_0} \]

\[ \int \vec{E} \cdot dA = \frac{1}{\varepsilon_0} \sum Q \]

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Gauss \rightarrow Coulomb

Calculate \( E \) of point like (+) charge \( Q \)

Consider sphere radius \( r \) centered at the charge.

Spherical symmetry: \( E \) is the same everywhere on the sphere, perpendicular to the sphere.

\[ \int \vec{E} \cdot dA = \int \vec{E} \cdot dA = E \cdot 4\pi r^2 \]

\[ \Phi = \frac{Q_{net}}{\varepsilon_0} \]

\[ E = \frac{Q}{4\pi \varepsilon_0 r^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]
Sinks and sources

• What goes in – goes out
  \( \Phi = \Phi_{\text{in}} + \Phi_{\text{out}} = 0 \)
  \( \Phi_{\text{in}} = -\Phi_{\text{out}} \)

• unless

• there is a source (positive charge)
  \( \Phi = \frac{Q_{\text{enc}}}{\varepsilon_0} > 0 \)

• or a sink (negative charge)
  \( \Phi = \frac{Q_{\text{enc}}}{\varepsilon_0} < 0 \)

What’s the flux?

• \( \Phi_1 = ? \)
• \( \Phi_2 = ? \)

Respect the symmetry

• Find electric field for the following configurations:
  Uniformly charged sphere. Charge \( Q \), radius \( R \).
  \[ r < R \rightarrow E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} \]
  \[ r > R \rightarrow E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \]

• Long uniform line of charge. Charge per unit length \( \lambda \).
  \[ E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r} \]

• Infinite plane of charge. Charge per unit of area \( \sigma \).
  \[ E = \frac{\sigma}{2\varepsilon_0} \]
Field near conductor

- Infinite plane of charge. Charge per unit of area $\sigma$
- Field inside conductor is zero, outside perpendicular to the surface.

$$E = \frac{\sigma}{\varepsilon_0}$$

Two parallel plates

- Infinite plates
- One positive, one negative, same charge density $\sigma$

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{\sigma}{\varepsilon_0}$$

Respect the symmetry

- Find electric field for the following configurations:
  - Uniformly charged sphere. Charge $Q$, radius $R$.
  $$r < R \rightarrow E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2} \quad r > R \rightarrow E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$$
  - Long uniform line of charge. Charge per unit length $\lambda$
  $$E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r}$$
  - Infinite plane of charge. Charge per unit of area $\sigma$
  $$E = \frac{\sigma}{2\varepsilon_0}$$