Physics 122

Electric potential, Systems of charges

Concepts

• Primary concepts:
  – Electric potential
  – Electric energy

• Secondary concepts:
  – Equipotentials
  – Electronvolt

Charges in electric fields

Positive charges experience force along the direction of the field
Negative charges – against the direction of the field.

$$\vec{F} = q\vec{E}$$
Potential electric energy

Just like gravity, electric force can do work. Work does not depend on the path; it depends only on the initial and final position. 

⇒ there is a potential energy associated with electric field.

Electric potential

$PE(q) \propto q$

- $PE/q$ is a property of the field itself—called electric potential $V$.

Electric potential

$V = \frac{PE}{q}$

- $V$ – electric potential is the potential energy of a positive test charge in electric field, divided by the magnitude of this charge $q$.
- Electric potential is a scalar (so much nicer!).
- Electric potential is measured in Volts ($V=J/C$).
- Potential difference between two points $\Delta V = V_b - V_a$ is often called voltage.
Charges in electric fields

\[ E = \text{const} \]

Force on charge \( q \):
\[ F = qE \]

Work done by the field to move this charge:
\[ W = Fd = dqE \]
\[ W = PE_a - PE_b = qV_a - qV_b = q\Delta V \]

\[ dE = \Delta V \]

\( E \approx \Delta V/d \), points from high potential to low

Sometimes electric field is measured in \( V/m \approx N/C \)

Non-uniform electric field

\[ \vec{E}(x) \Rightarrow \vec{F}(x) = q\vec{E}(x) \]
\[ W = \int \vec{F}dx = q\int \vec{E}dx \]
\[ U_a - U_b = -W_{ab} \]
\[ V = \int \vec{E}d\vec{a} \]

Determine \( E \) from \( V \)

- Think ski slopes
- If \( V \) depends on one coordinate \( x \)
  - \( E \) is directed along \( x \) from high \( V \) to low
  \[ E(x) = -\frac{dV}{dx} \]
- If \( V \) depends on \( x, y, z \)
  \[ E_x = -\frac{\partial V}{\partial x}; \quad E_y = -\frac{\partial V}{\partial y}; \quad E_z = -\frac{\partial V}{\partial z} \]
Electric field and potential in conductors

\[ E = 0 \text{ in good conductors in the static situation.} \]
\[ E \] is perpendicular to the surface of conductor.

Metal hollow boxes are used to shield electric fields.

When charges are not moving conductor is entirely at the same potential.

\[ E = E_{\text{external}} + E_{\text{internal}} = 0 \]
\[ V = \text{const} \]

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Electronvolt

• Energy that one electron gains when being accelerated over 1V potential difference is called electronvolt eV:
  \[ 1eV = 1.6 \times 10^{-19} \text{C} \]
  \[ 1V = 1.6 \times 10^{-19} \text{J} \]

• Yet another unit to measure energy,

• Commonly used in atomic and particle physics.

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Equipotentials

Equipotentials

• are surfaces at the same potential;
• are always perpendicular to field lines;
• Never cross;
• Their density represents the strength of the electric field
• Potential is higher at points closer to positive charge
Potential of a point charge

Potential \( V \) of electric field created by a point charge \( Q \) at a radius \( r \) is

\[
V(r_0) = -\int Edr = -kQ \int_0^r \frac{dr}{r^2} = k \frac{Q}{r_0}
\]

\( Q > 0 \Rightarrow V > 0 \)
\( Q < 0 \Rightarrow V < 0 \)

Do not forget the signs!

Potential goes to 0 at infinity.

Equipotentials of a point charge are concentric spheres.

Superposition of fields

**Principle of superposition:**

Net potential created by a system of charges is a **scalar (!) sum** of potentials created by individual charges:

\[
V = V_1 + V_2 + V_3 + \ldots
\]

Potential is a scalar \( \Rightarrow \) no direction to worry about.

Electric Dipole potential

\[
V(r) = k \frac{Q}{r} - k \frac{Q}{r + \Delta r} = kQ \frac{\Delta r}{r(r + \Delta r)}
\]

\( \Delta r = \cos \theta \cdot \Delta r \)

\( \text{Let } r \gg L, \)

\[
V(r) = k \frac{p \cos \theta}{r^2}
\]
Test problem

- What is wrong with this picture?
  - A Equipotentials must be parallel to field lines
  - B Field lines cannot go to infinity
  - C Some field lines point away from the negative charge
  - D Equipotentials cannot be closed

The electric potential of a system of charges

\[ V = V_1 + V_2 + V_3 + \ldots \]
\[ V_i = \frac{k \cdot q_i}{r_i} \text{ - distance from charge } i \text{ to point in space where } V \text{ is evaluated} \]

\[ dV = k \frac{dq}{r} \]
\[ V = \int dV \]
Symmetry and coordinate systems

• Coordinate systems are there to help you
• You have a choice of
  – System type
    • Cartesian
    • Cylindrical
    • Spherical
  – Origin (0,0), Direction of axis
• A good choice (respecting the symmetry of the system) can help to simplify the calculations

Ring of charge

• A thing ring of radius \( a \) holds a total charge \( Q \). Determine the electric field on its axis, a distance \( x \) from its center.

\[
E = k \frac{Qx}{(x^2 + a^2)^{3/2}}
\]

Ring of charge

• A thing ring of radius \( a \) holds a total charge \( Q \). Determine the electric potential on its axis, a distance \( x \) from its center.

\[
dV = k \frac{Q d\phi}{2\pi \sqrt{x^2 + a^2}}
\]

\[
V = k \frac{Q}{(a^2 + x^2)^{1/2}}
\]
**Work to move a charge**

How much work has to be done by an external force to move a charge $q=+1.5 \, \mu C$ from point a to point b?

**Work-energy principle**

$$W = \Delta KE + \Delta PE = PE_2 - PE_1$$

$$PE_2 = qV_2 = q(V_1^b + V_2^b)$$

$$PE_1 = qV_1 = q(V_1^a + V_2^a)$$

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**E near metal sphere**

- Find the largest charge $Q$ that a conductive sphere radius $r=1cm$ can hold.
- Air breakdown $E=3x10^6 V/m$

$$E(r) = -\frac{dV}{dr} = -k\left(\frac{1}{r}\right) = k \frac{Q}{r^2}$$

$$Q = \frac{1}{k} E_r \cdot 2 = \frac{3 \times 10^6}{9 \times 10^{-9}} (10^{-2})^2 = 0.33 \times 10^{-7} = 0.033 \, \mu C$$

Larger spheres can hold more charge