AC circuits

Physics 122

11/2/12 Lecture XIX

LC circuit

• Two forms of energy:
  - Electric – in a capacitor
  - Magnetic - in solenoid

• Oscillator!
  - When energy in the capacitance is at its maximum, energy in the inductance is at its minimum
  - No energy is lost, it is just changing its form

\[ U_{\text{total}} = \frac{1}{2} CV^2 + \frac{1}{2} LI^2 = \text{const} \]

• Total drop of voltage over a closed circuit must be 0:

\[ 0 = V_L + V_C = Q \frac{C}{L} + \frac{L}{C} \frac{dI}{dt} + \frac{1}{C} \int I \, dt + L \frac{dI}{dt} \]

• Take derivative over time:

\[ 0 = \frac{1}{C} I + L \frac{d^2I}{dt^2} \Rightarrow \frac{d^2I}{dt^2} = -\frac{1}{L} \frac{1}{C} I \]

• Second derivative of function is proportional to the negative function

\[ I = I_0 \sin(\omega t) \]

\[ \omega = \frac{1}{\sqrt{LC}} \]
LCR circuit

- Resistors dissipate energy
  - Convert electrical energy into thermal energy
- Resistor acts like friction for a weight on a spring
- Damped oscillator!

Inductance

- Magnetic field in a solenoid
  - Magnetic flux through \( B \propto I \)
- It creates a magnetic flux through itself
  \[ \Phi = BA \]
  \[ \frac{d\Phi}{dt} = \frac{dB}{dt} \frac{dl}{dt} \]
- Changing magnetic flux generates emf (voltage gained) = drop of Voltage (voltage lost)
  \[ \text{emf} = -V_L = -N \frac{d\Phi}{dt} = -L \frac{dl}{dt} \]

AC circuits

- External source of alternating voltage \( V(t) = V_0 \sin \omega t \) – Forced oscillator
- Frequency \( f \), cyclic frequency \( \omega = 2\pi f \)
- Active elements
  - Inductors
  - Capacitors
- Passive elements
  - Resistors
Phasor diagram

- Graphical way to present current and voltage in AC circuit
- Present current and voltage in a certain element (resistor, inductance, capacitance) as vectors rotating in (x,y) plane
- Vector length is equal to maximum possible value of I or V: I₀ and V₀
- Vector projection on y-axis is equal to the value of current or voltage at the moment t

Current and voltage in AC circuit

- Drop of voltage over resistor (V) follows I
- Current and voltage in phase

Current and voltage in AC circuit

- Inductor: current lags voltage
Current and voltage in AC circuit

\[ V_C = \frac{Q}{C} = \frac{1}{C} \int I \, dt \]
\[ V_C = \frac{1}{C} \int I_0 \sin(\omega t) \, dt = -\frac{1}{\omega C} I_0 \cos(\omega t) \]
\[ V_0 = \frac{1}{\omega C} I_0 = X_L I_0 \]
\[ X_C = \frac{1}{\omega C} \]

Capacitor: voltage lags current

LCR circuit

\[ I(t) = I_0 \sin(\omega t) \]
\[ V_{L_0} = I_0 R \]
\[ V_{iL} = I_0 L \]
\[ V_{iC} = I_0 X_C \]
\[ R = R \]
\[ X_L = \omega L \]
\[ X_C = \frac{1}{\omega C} \]

Ohm's law!!

\[ V^* = \overrightarrow{V}_x + \overrightarrow{V}_y + \overrightarrow{V}_z = ZI \]
\[ Z \text{ - impedance} \]

Impedance

\[ V_0 = Z I_0 \]
\[ |Z| = \sqrt{R^2 + (X_L - X_C)^2} \]
\[ \tan \phi = \frac{(X_L - X_C)}{R} \]

This equation relates maximum voltage to maximum current.
Mind, that they do not happen at the same time. There is a phase shift between \( I_{\text{max}} \) and \( V_{\text{max}} \). 

\[ \phi \]
Phase angle and power factor

\[ \tan \varphi = \frac{X_L - X_C}{R} \]

\[ \cos \varphi = \frac{R}{Z} \]

\[ V_{rms} = ZI_{rms} \]

\[ P = V \cdot I = V_{rms} I_{rms} \cos \varphi \]

Cos \( \varphi \) – power factor

NB. \( V \) and \( I \) are not real vectors, only in phasor diagram representation

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Energy in AC circuit

Total average power in AC

\[ P = V_{rms} I_{rms} \cos \varphi \]

Power dissipated only in resistors:

\[ P = I_{rms}^2 R \]

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Resonance in LCR circuit

\[ I_{rms} = \frac{V_{rms}}{Z} \]

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

• Largest current when impedance is the smallest
• \( X_L, X_C \) are functions of frequency
• At a certain frequency

\[ \begin{align*}
\omega L &= \frac{1}{\omega C} \\
\omega &= \frac{1}{\sqrt{LC}} \\
\omega &= \frac{1}{2\pi\sqrt{LC}}
\end{align*} \]

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