Questions 1-4 (20 points total) deal with the situation sketched at right. Three charges of +6μC are placed at three corners of a square 0.10 m on a side.

1. (6 points) Calculate the magnitude of the electric field at the forth corner (point A).

\[ \mathbf{E}_1 = \mathbf{k} \frac{Q}{d^2} = E_1 \]
\[ a = 0.1 \text{ m} \]
\[ E_A = ? \]
\[ |\mathbf{E}_1 + \mathbf{E}_3| = E_1 \cdot \sqrt{2} = \mathbf{k} \frac{Q \sqrt{2}}{a^2} \]
\[ \mathbf{E}_2 + \mathbf{E}_1 + \mathbf{E}_3 = \mathbf{k} \frac{Q}{a^2} \left( \sqrt{2} + \frac{1}{2} \right) = 9 \cdot 10^9 \cdot 6 \cdot 10^{-6} \left( 1.4 + 0.5 \right) = 102.6 \cdot 10^{-6} + 2 = 1.07 \text{ V/m} \]

2. (2 points) If a positive charge is placed at point A and then released it will move towards point B at the center of the square.

3. (6 points) Calculate the electric potential at point A?

\[ V_A = V_1 + V_2 + V_3 = \mathbf{k} \frac{Q}{a} + \mathbf{k} \frac{Q}{a \sqrt{2}} + \mathbf{k} \frac{Q}{a} = \mathbf{k} \frac{Q}{a} \left( 2 + \frac{1}{\sqrt{2}} \right) = 9 \cdot 10^9 \cdot 6 \cdot 10^{-6} \left( 2 + \frac{1}{1.4} \right) = 146 \cdot 10^9 - 6 + 1 = 1.5 \cdot 10^6 \text{ V} \]

4. (6 points) Calculate the work done by an external force to move +0.1 μC charge from point A to point B

\[ V_B = 3 \cdot \mathbf{k} \frac{Q}{a} \left( \frac{V_2}{2} \right) = 116 \cdot 10^4 = 1.2 \cdot 10^6 \text{ V} \]
\[ W = P_E_B - P_E_A = q \left( V_B - V_A \right) = -10^{-7} \cdot (1.5 - 1.2) \cdot 10^6 = -0.037 \text{ J} \]
Questions 5-8 (17 points total) deal with the situation shown at right. A thin cylindrical shell of radius $R_1$ is surrounded by a second cylindrical shell of a radius $R_2$. The inner shell has a positive charge per unit length of $\lambda$ ($\lambda = Q/L$) and the outer shell carries negative charge of the magnitude per unit length $-\lambda$. Assume the length $L$ of the shells to be much greater than the outer radius.

5. (4 points) Evaluate the electric field as a function of radius $r$ for $r < R_1$.

\[
\oint q_{encl} = 0 \Rightarrow E = 0 \quad r < R_1
\]

\[
\oint E \cdot dA = E \cdot 2\pi R \cdot L
\]

6. (6 points) Evaluate the electric field as a function of radius $r$ for $R_1 < r < R_2$.

\[
E \cdot 2\pi R \cdot \lambda = \frac{\lambda_1 \Delta}{\varepsilon_0}
\]

\[
E = \frac{\lambda_1}{2\pi \varepsilon_0 R} = \frac{R_1}{R} < r < R_2
\]

\[
E = + \frac{\lambda}{2\pi \varepsilon_0 R}
\]

7. (4 points) Evaluate the electric field as a function of radius $r$ for $r > R_2$.

\[
\oint q_{encl} = 0 \quad E = 0 \quad r > R_2
\]

8. (3 points) A positive charge is placed between the shells it is likely to move
(a) toward the outer shell  
(b) toward the inner shell.  
(c) clockwise.
Question 9-11. (18 points total) Five different light bulbs with resistances of 1.0, 2.0, 3.0, 4.0 and 5.0 Ohm are connected in parallel to a 12V source through a pair of leads of total resistance of 1.0 Ohm.

9. (6 points) Find the equivalent resistance of the circuit.

\[ R_{\text{equiv}} = \frac{1}{R_1 + R_2 + R_3 + R_4 + R_5} = \frac{1}{1 + 2 + 3 + 4 + 5} = \frac{1}{15} \]

10. (6 points) Find the power dissipated in the leads.

\[ I = \frac{V}{R_{\text{tot}}} = \frac{12}{14.4} = 0.833A \]

\[ P_{\text{dis}} = I^2 \cdot R_{\text{lead}} = 6.9W \]

11. (6 points) Find the current through the 5.0 Ohm light bulb.

\[ V_{AB} = V_0 - I \cdot R_{\text{lead}} = 12 - 8.31 = 3.7V \]

\[ I_5 = \frac{V_{AB}}{R_5} = 0.744A \]

12. (8 points) An amateur radio operator wishes to receive radio signals with a wavelength of 5.8m. What value of inductance should she connect in series with a 250 pF \((250 \times 10^{-12} \text{ F})\) capacitor to tune in these signals?

\[ \lambda = 5.8 \text{ m} \]

\[ f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{5.8 \text{ m}} = 52 \times 10^8 \text{ Hz} \]

\[ \omega = 2\pi f = 3.25 \times 10^8 \text{ rad/s} \]

Resonance frequency of the circuit

\[ \omega = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{3.25^2 \times 10^8 \times 2.5 \times 10^{-10}} \]

\[ = 3.8 \times 10^{-2} \text{ H} = 3.8 \times 10^{-8} \text{ H} \]
\[ \frac{U}{2} = \frac{\epsilon_0 V^2}{2} \]

\[ C = \frac{\epsilon_0 A}{d} \]

\[ V = \text{constant} \]

13. (3 points) A dielectric with dielectric constant \( K = 2 \) is inserted in a capacitor while it remains connected to the battery. The energy of the capacitor is
   a) doubled    b) quadrupled    c) halved    d) divided by 4.

14. (2 points) It is possible to make two resistors of equal value from equal lengths of copper and aluminum wires.

Questions 15-16. (14 points total) A beam of electrons passes without being deflected through crossed electric and magnetic fields of magnitude \( 8.8 \times 10^3 \text{V/m} \) and \( 3.5 \times 10^{-3} \text{T} \) respectively.

15. (7 points) What is the velocity of the electron beam?

\[ F_E + F_B = 0 \]

\[ |F_E| = |F_B| \]

\[ eE = qVB \]

\[ V = \frac{E}{B} = \frac{8.8 \times 10^3}{3.5 \times 10^{-3}} = 2.5 \times 10^6 \text{ m/s} \]

16. (7 points) What is the radius of the electron orbit if the electric field is turned off?

\[ E = 0 \quad F_E = 0 \quad F_B = qVB \quad F \perp \nu \]

\[ F = ma = m \frac{v^2}{R} \quad \text{centripetal acceleration} \]

\[ qVB = \frac{m v^2}{R} \]

\[ R = \frac{m \nu}{qB} = \frac{9.11 \times 10^{-31} \times 2.5 \times 10^6}{1.6 \times 10^{-19} \times 3.5 \times 10^{-3}} \approx \]

\[ = 4.1 \times 10^{-31} \text{m} = 4.1 \text{ mm} \]
Questions 17-19 (18 points total) deal with this AC series circuit.
The circuit is connected to a 60.0V (rms) AC generator with
frequency 15kHz (15\times 10^3 \ Hz). The resistor is 85 ohms, the
capacitor is 60\text{nF}
(60\times 10^{-9}\text{F}), and the inductance is 3.5 mH(3.5 \times 10^{-3}\text{ H}).

17. (6 points) Find the impedance of the circuit.
\[ f = 15 \cdot 10^3 \text{Hz} \quad \omega = 2\pi f = 94 \cdot 10^3 \text{rad/s} \]
\[ R = 85 \Omega \]
\[ C = 60\text{nF} = 60 \cdot 10^{-9} \text{F} \quad X_C = \frac{1}{\omega C} = 1.8 \cdot 10^{-3} \text{rad} = 180 \Omega \]
\[ L = 3.5 \cdot 10^{-3} \text{H} \quad X_L = \omega L = 94 \cdot 10^{-3} \cdot 3.5 \cdot 10^{-3} = 329 \Omega \]
\[ Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{85^2 + (329 - 180)^2} = 171 \Omega \]

18. (6 points) Find the phase angle.
\[ \varphi > 0 \quad \tan \varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \]
\[ = \frac{329 - 180}{85} = 1.75 \]
\[ \varphi = 60^\circ \]

19. (6 points) What is the maximum drop of voltage over L?
\[ V_L_{\text{max}} = I_{\text{max}} \cdot X_L \]
\[ I_{\text{max}} = I_{\text{RMS}} \cdot \sqrt{2} \]
\[ I_{\text{RMS}} = \frac{V_{\text{RMS}}}{Z} = \frac{60}{171} = 0.35 \text{A} \]
\[ V_L_{\text{max}} = 329 \cdot 0.49 = 162 \text{V} \]
\[ I_{\text{max}} = 0.49 \text{A} \]
Question 20-22 (extra credit) (15 points total=6% of the total course grade). A flat slab of nonconducting material carries a uniform positive charge per unit volume \( \rho > 0 \text{C/m}^3 \). The slab has thickness \( 2a \), which is small compared to the height and breadth of the slab. The slab is oriented perpendicular to x-axis so that \(-a < x < a\). An alpha particle (charge \( +2e \), mass=1 m,) moving along the x-axis enters the slab through a small hole drilled through the entire slab along x-axis.

20. (5 points) Find the electric field inside the slab as a function of \( x \).

\[
\text{Field is symmetric around } x = 0 \text{ plane. Gauss law: } \quad \oint A \cdot E = \frac{\rho}{\varepsilon_0} \cdot A \]

\[
E = \frac{\rho}{\varepsilon_0} \cdot x
\]

21. (5 points) Find the work done by the electric field as a function of \( x \).

\[
W_E = \int_{-a}^{a} F \cdot dx = \int_{-a}^{a} 2e \cdot E \cdot dx = \frac{2e \rho}{\varepsilon_0} \int_{-a}^{a} x \, dx
\]

\[
= \frac{e \rho}{\varepsilon_0} \left[ \frac{x^2}{2} \right]_{-a}^{a} = \frac{e \rho}{\varepsilon_0} \left( a^2 - (-a)^2 \right) = \frac{2e \rho a^2}{\varepsilon_0}
\]

22. (5 points) Find the critical velocity \( v \) with which the alpha particle enters the slab such that above this velocity the particle exits on the opposite side and below which the particle exits on the same side where it entered.

Work-energy \( W = \Delta KE \)

if \( \frac{mv^2}{2} > -W_E(0) \) - enough energy to go over \( x = 0 \) plane

if \( \frac{mv^2}{2} < -W_E(0) \) - shoots back

\[
\frac{mv^2}{2} = \frac{e \rho a^2}{\varepsilon_0}
\]

\[
\therefore \quad v_{\text{crit}} = \frac{\sqrt{2e \rho a^2}}{m \varepsilon_0}
\]